THE JOURNAL of the Acoustical Society of America

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The Acoustical Work of the National Physical Laboratory*

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(Received November 22, 1935)

THE National Physical Laboratory which is situated some 12 miles southwest of London is a state institution devoted to research and testing, much on the lines of the National Bureau of Standards. The study of acoustics at the National Physical Laboratory dates from about 1922 when it became one of the activities of the Physics Department of the Laboratory. The developments in the industrial applications of acoustics since that time—in the cinema and in the gramophone and radio industries, and in architectural acoustics—together with the remarkable growth of public interest in the suppression of noise, have resulted in a steadily increasing call upon the Laboratory for acoustical tests and investigations of all kinds.

The acoustical work of the Laboratory may be divided into two broad fields. The first is concerned with the various aspects of the acoustics of buildings, including the determination of the absorption coefficients of materials used for the control of reverberation, and the measurement of the sound insulation values of building structures. The second field, based fundamentally on the absolute measurement of sound, includes measurements of the performance of various types of acoustical instruments and the measurement of noise.

Accommodation specially designed for acoustical work was not at first available but the first section of a new Acoustics Laboratory was completed in 1933 and is now in active use.[1]

THE ACOUSTICS LABORATORY

The Acoustics Laboratory (Fig. 1) is situated in a quiet site, well removed from roads carrying traffic. Fig. 2 shows a sectional plan and elevation. The completed portion of the building comprises a reverberation room and a pair of transmission rooms, each with its own measurement room for the operator and apparatus. It is hoped that the parts shown hatched on the plans, an upper transmission room for the measurement of the sound insulation value of floors and a large heavily lagged room for fundamental sound measurements, will shortly be provided. Complete acoustical isolation is aimed at in the case of the experimental rooms, the enclosing walls, floor and ceiling being of massive masonry

*Note: This article is one of a series of papers describing the acoustic work being done in different countries. Later papers may be expected. THE EDITOR.

[1] G. W. C. Kaye. "The New Acoustics Laboratory at the National Physical Laboratory," Nature **134**, 202 (1934).

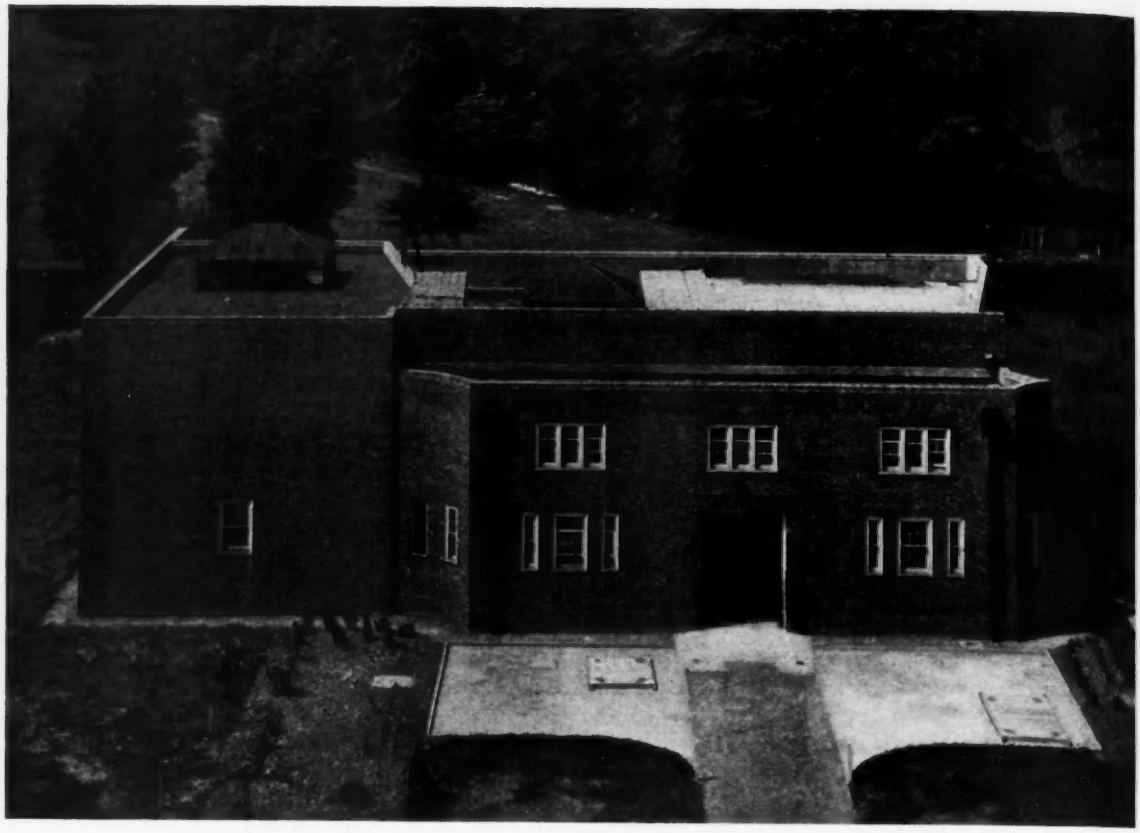

FIG. 1. The Acoustics Laboratory.

and everywhere double. The inner shell of each room (14 inches thick) is completely independent of the outer (9 inches thick) and rests only on insulating piers on separate foundations. The insulation in the piers is slab cork (about 3 inches thick) and provision has been made for the hydraulic lifting of the inner rooms (which weigh 150–200 tons), so that should the cork deteriorate under the sustained heavy load it can be renewed.

The ventilation of the building is by pressure feed, and constant-temperature heating in the experimental rooms is aimed at by passing warmed air between the double walls. Concentric cable is used in the wiring of the equipment, and all electric supplies are led into the experimental rooms by means of flexible leads across the gaps between the double walls. When desired, the entire building or any selected section of it can be rendered electrically isolated by cutting off all external power supplies, requirements being then met from a battery.

SOUND ABSORPTION COEFFICIENTS

The measurement of sound absorption coefficients received early attention at the Laboratory. Methods were developed for the determination of absorption coefficients, both under conditions of normal incidence (the stationary wave method)[2] and under reverberant conditions. The latter value is of course the figure required in auditorium calculations, but the former method, although of very restricted application, has occasionally proved useful when information of a preliminary and comparative nature would serve the purpose. Sound absorption coefficients had been determined by the reverberation method in temporary accommodation for some years prior to the erection of the new reverberation room. The greatly improved

[2] A. H. Davis and E. J. Evans, "Measurement of Absorbing Power of Materials by the Stationary Wave Method," Proc. Roy. Soc. **A89**, 127 (1930).

facilities provided by the latter have enabled the accuracy of measurements to be increased and the range of frequencies covered to be extended. The new reverberation room shown in Fig. 3, has a volume of about 10,000 cubic feet, and is asymmetric both in plan and elevation. The walls are not parallel, nor is the ceiling horizontal, thus assisting in minimizing the influence of the natural transverse resonances of the air in the room, as well as promoting the uniform and random distribution of sound which the several reverberation formulae postulate. The closely fitting double entrance doors are each of solid steel, the inner being $3\frac{5}{8}$ inches thick and $2\frac{1}{2}$ tons in weight, and the outer $2\frac{1}{2}$ inches thick and over $1\frac{1}{2}$ tons in weight, each corresponding in superficial weight to the adjacent walls. The shutters to the skylight, which are of similar construction, are electrically operated from below. The floor is of painted concrete and the walls and ceiling are finished in painted hard plaster. The Sabine reverberation period for a note of 500 cycles per second is about 13 seconds when the room is empty. In one of the walls are $\frac{3}{4}$-inch steel swing doors covering a shallow recess in which may be mounted 100 square feet of the material of which the sound absorption coefficient is required. Alternatively, in appropriate cases, the test material may be laid on the floor.

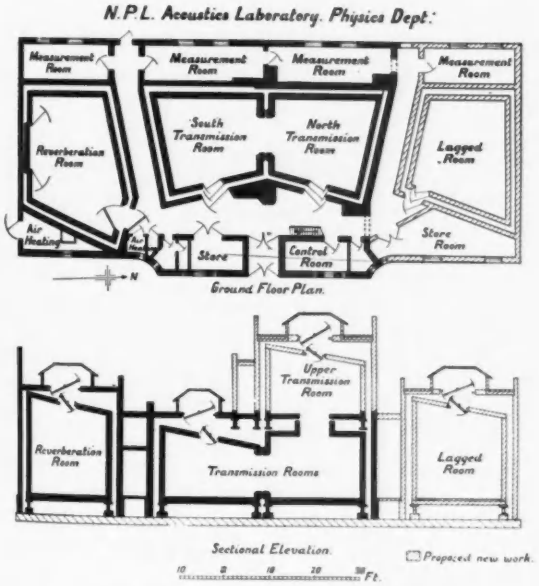

FIG. 2. Sectional plans of the Acoustics Laboratory.

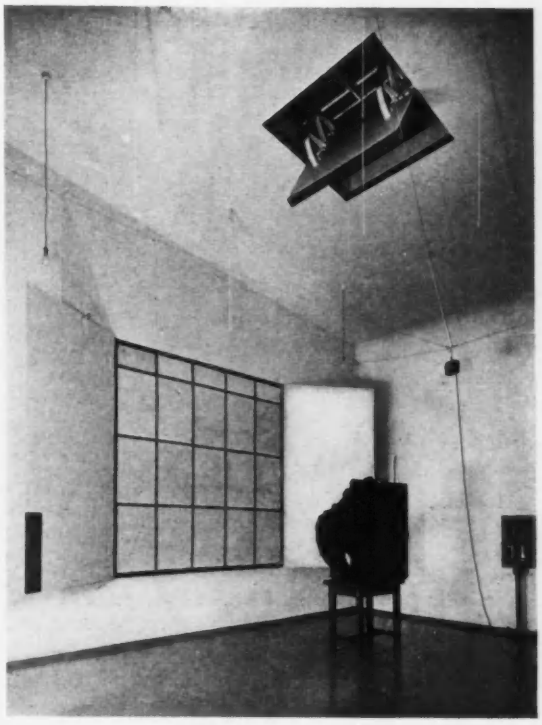

FIG. 3. Reverberation chamber, showing loudspeaker, microphone and test panel of which the sound absorption coefficient is required.

Absorption coefficients are usually determined on the basis of Eyring's formula from measurements of the respective rates of decay of reverberant sound in the room when empty (or with the test material covered with the steel doors), and with the test material exposed. The rates of decay are measured instrumentally. A loudspeaker is used as a source of sound, a warble tone being employed in order still further to promote the even distribution of sound throughout the room. The reverberant sound is picked up by a microphone, the e.m.f. from which is led to an amplifier of variable gain in the adjacent measurement room. The act of switching off the loudspeaker automatically starts an electric chronoscope, which is stopped by the action of a thyratron relay when the output of the microphone amplifier has fallen to a predetermined value. When the gain of the amplifier is increased by say 10 decibels the reverberant sound must decay a further 10 decibels before the chronoscope is stopped, and the difference in the two readings of the chronoscope gives the time of

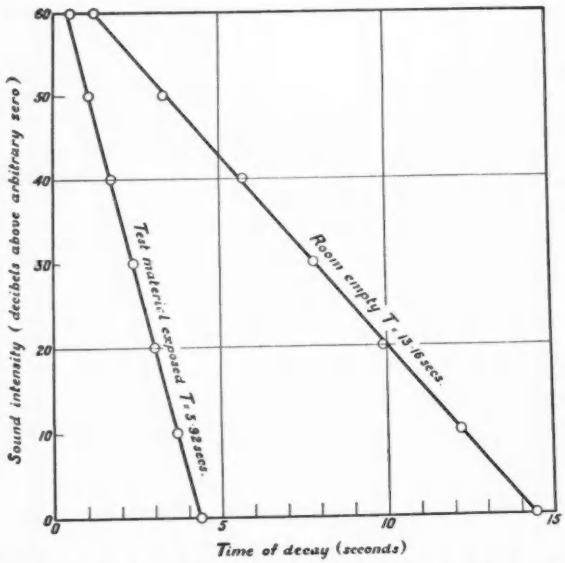
Sound intensity (decibels above arbitrary zero)
Time of decay (seconds)
Test material exposed T = 5.92 secs.
Room empty T = 13.16 secs.

FIG. 4. Decay of sound intensity in reverberation chamber. Frequency, 500±50 cycles per second. Absorption coefficient of material 0.85.

decay over this interval. By further increasing the gain of the amplifier by successive steps of 10 decibels, the rate of decay of the reverberant sound may be measured over a total range up to 60 decibels or more, the high degree of acoustic isolation of the room rendering this readily possible. Measurements are normally made at octave frequencies from 125 to 8000 cycles per second, some four positions of the microphone being used at each frequency. Fig. 4, showing practically uniform rates of decay over a range of 60 decibels, both with the room empty and with 100 sq. ft. of a test material introduced, is typical of the results obtained at frequencies of 500 cycles per second and above. At lower frequencies the rate of decay becomes progressively less uniform.

A steady demand from industry has been received for absorption measurements of this nature for a number of years and the tests have included a wide range of materials, e.g., acoustic plasters and tiles, fiber boards, acoustic felts, slag wool, glass silk, etc., with absorption coefficients ranging from below 0.05 to above 0.9. In 1933 the Laboratory using the temporary accommodation then available took part in an international comparison of measurements of absorption coefficients organized by the Acoustical Society of America.[3] The test material used has recently been retested in the new reverberation room with substantially the same results. It is hoped that the enquiry will throw light on the dependence of absorbing power on the size, shape and distribution of a material.

SOUND TRANSMISSION

Walls

The development of methods for measuring the insulation value of walls, partitions, etc.,

[3] See P. E. Sabine, "Measurement of Sound Absorption," J. Acous. Soc. Am. **6**, 239 (1935).

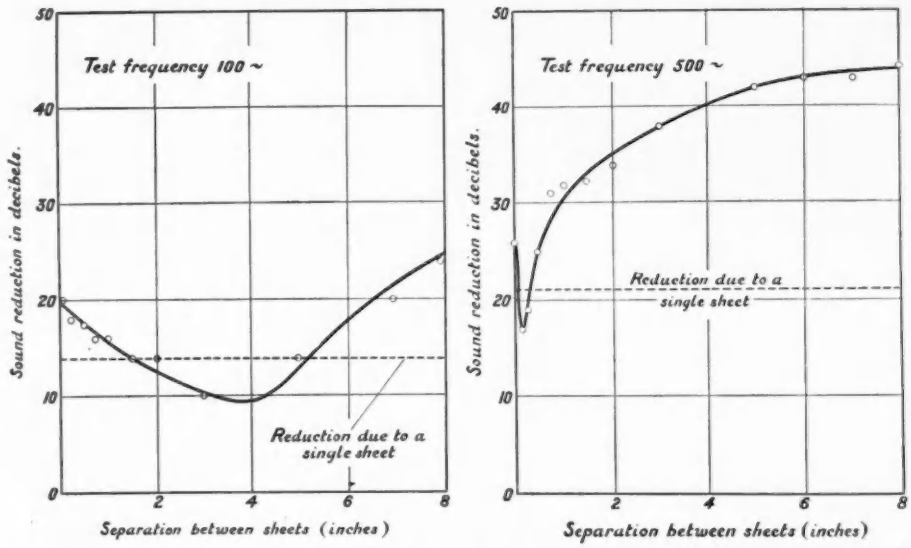
Test frequency 100 ~
Sound reduction in decibels.
Reduction due to a single sheet
Separation between sheets (inches)
Test frequency 500 ~
Sound reduction in decibels.
Reduction due to a single sheet
Separation between sheets (inches)

FIG. 5. The influence of the spacing of the components upon the noise insulating value of a double window of 21-oz. glass showing a position of minimum insulation.

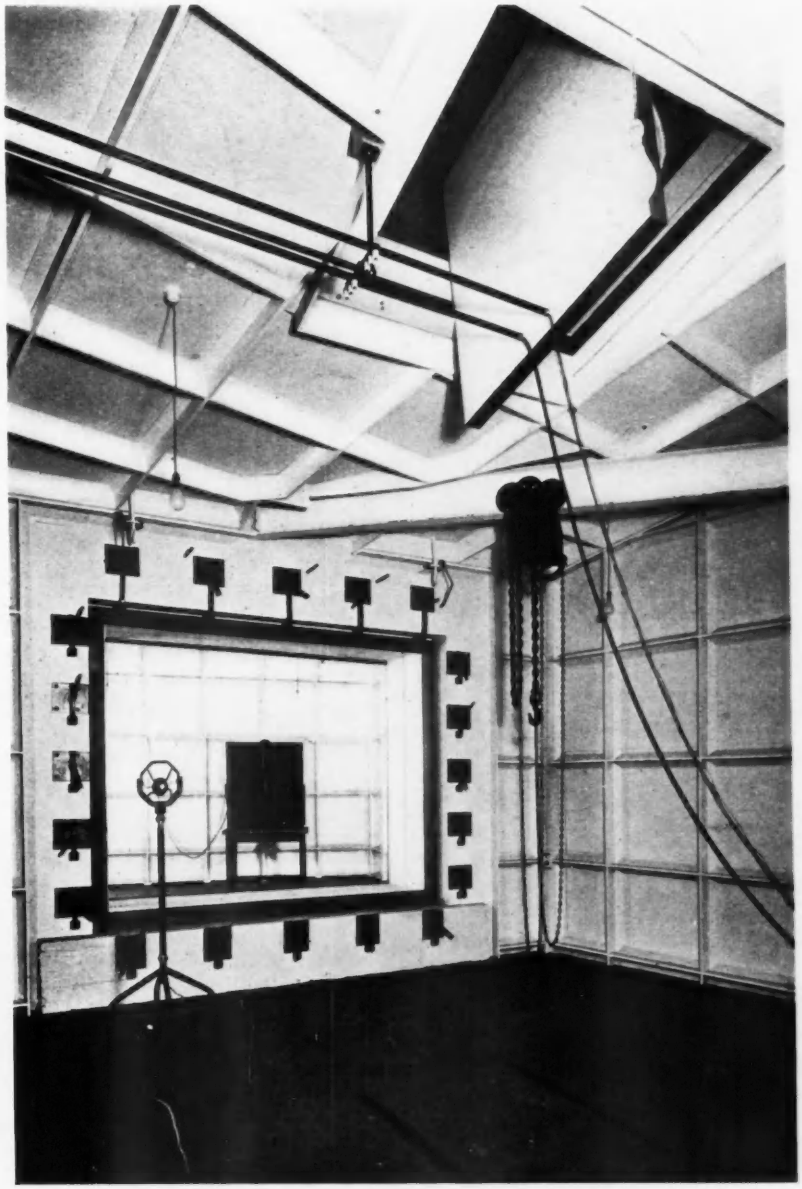

FIG. 6. Transmission rooms, showing test aperture.

against air-borne sounds was begun in 1924 in accommodations consisting of two small rooms separated by a double wall in which was an aperture some 5 feet by 4 feet for the accommodation of the test specimen. The rooms were heavily lined with sound absorbent, and in one a beam of sound was directed obliquely onto the aperture. The sound reduction factor of the test partition was determined from measurements of the sound received in the second room both with and without the partition in position. The fundamental dependence of the sound reduction of a single homogeneous partition on its superficial weight was established for a range of materials from paper to a $4\frac{1}{2}$-inch brick wall.[4, 5]

[4] A. H. Davis and T. S. Littler, "The Transmission of Sound Through Partitions. I. Felt-Like Materials," Phil. Mag. **3**, 177 (1927).

[5] A. H. Davis and T. S. Littler, "The Transmission of Sound Through Partitions. II. Vibrating Partitions," Phil. Mag. **7**, 1050 (1929).

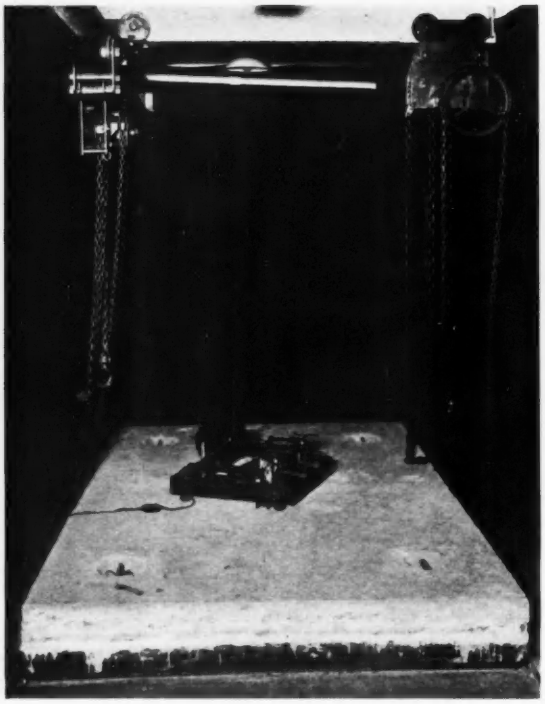

FIG. 7. Floor testing site, showing impact machine in position on test floor.

A good deal of work has been carried out on composite partitions. Properly designed double partitions, for example, may be more sound proof than a single partition of the same total weight. With improper spacing, however, the sound reduction may actually be less than that of a single partition of the same weight.[6] The effect is illustrated in Fig. 5.

The new transmission rooms shown in Fig. 6, provide facilities for measurements on specimens of larger size and of greater insulating efficiency. The two rooms are on independent foundations and are of the same irregular shape and general construction as the reverberation room, except that the entrance doors are triple and of solid wood 3 inches thick. The test aperture will take specimens up to about 10 feet by 8 feet, which are either erected *in situ* or assembled in a framework elsewhere and then mounted in position by the aid of the clamps shown. A runway is provided for transporting heavy specimens.

[6] J. E. R. Constable, "The Acoustical Insulation Afforded by Double Partitions Constructed from Similar Components," Phil. Mag. **18**, 321 (1934).

At present, measurements are made with the rooms in a reverberant condition giving random incidence of sound on the test partition, but provision has also been made for lining the rooms with absorbents when a beam of sound is used. A loudspeaker in one room produces a warble tone, and the sound insulation value of the partition is determined from measurements of the average sound intensity in each room both with and without the partition in the aperture. The average sound intensity is determined from microphone measurements at several different positions. The position of the microphone is controlled at will from the adjacent measurement room, where the oscillator actuating the loudspeaker and the microphone amplifier are situated. Measurements are normally made at a series of frequencies from 100 to 4000 cycles per second.

The transmission rooms can readily be adapted for testing the sound attenuation in ventilation ducts and silencers, as well as for studying the quieting of machinery.

Floors

Pending the provision of the upper transmission room (see Fig. 2) temporary arrangements (shown in Fig. 7) have been devised for measuring the comparative insulation afforded by floor structures against impact sounds. The specimen floor (some 8 feet by 5 feet) is bedded on a heavy ferro-concrete frame which stands over an aperture in a 6-inch reinforced concrete floor. The frame rests upon a strip of rubber which insulates it from the aperture surround and makes an airtight seal. A succession of blows, corresponding in energy to average footfalls, is made on the test specimen by a motor-driven impact machine comprising four hammers operated by cams. Two types of hammer head are employed, one hard (of Keramot) and the other soft (of rubber). The loudness of the noise produced below the floor is measured subjectively by a group of observers. Many types of composite floors have already been tested. Work of this nature is also being carried out in conjunction with the Building Research Station where temporary facilities have been provided to accommodate floor specimens of larger size (18 feet by 15 feet).

The Laboratory findings on both walls and floors are frequently implemented by "field

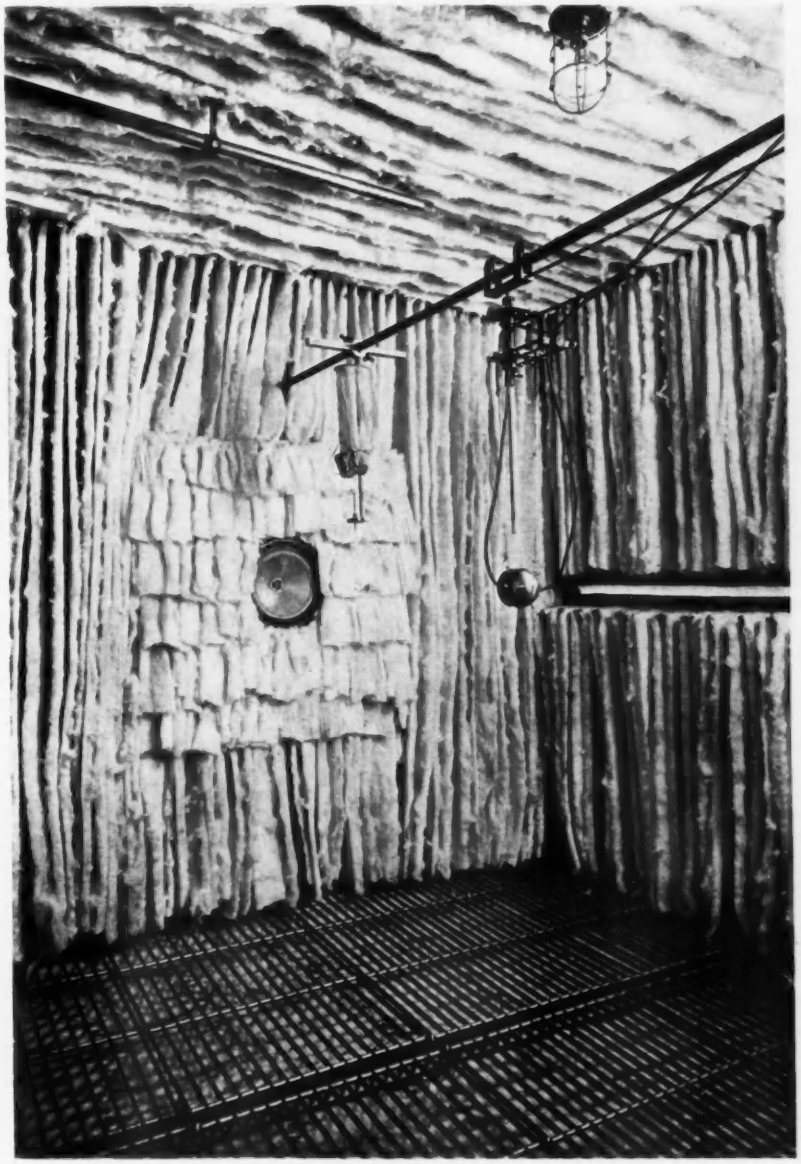

FIG. 8. Absorbent-lined cabinet for absolute measurement of sound.

tests" on buildings actually erected. An extended series of investigations on building structures is in progress for Government Departments, and the demand for tests from private firms has in recent years been so great that it has been necessary to continue to utilize the smaller transmission rooms as well as the large.

ACOUSTICS OF AUDITORIUMS

The Laboratory undertakes work in connection with the provision of good acoustical conditions in auditoriums, both in existing buildings, which have proved defective in this respect, and in advance of construction. Apparatus for photographing the progress of a sound pulse in model sections of buildings has been developed, and used in elucidating the sources of possible echoes, and for purposes of illustration.[7, 8] Investigations

[7] A. H. Davis and N. Fleming, "Sound Pulse Photography as Applied to the Study of Architectural Acoustics," J. Sci. Inst. **3**, 393 (1926).

[8] N. Fleming, "The Photography of Sound Waves," Proc. Seventh Int. Cong. Phot. 318 (1928).

have been carried out for government and other public bodies, as well as for private firms, both in this country and abroad.

Absolute Sound Measurements

The absolute measurement of sound is of fundamental importance for the calibration of microphones and their subsequent use for sound measurements of all kinds. The methods employed at the National Physical Laboratory are based primarily upon the use of the Rayleigh disk, which measures sound particle velocity. Microphones, of the condenser type, calibrated in absolute terms, are maintained as laboratory standards against which other microphones may be calibrated by comparison under appropriate conditions.

For the fundamental calibration of the standard microphones thin circular glass disks of 1 cm diameter, silvered on one side are normally employed. The disks are suspended from their edges by fine quartz fibers of which the torsion constants are previously measured, so that the particle velocity may be calculated from the deflection produced.

For the determination of the field sensitivity of the standard microphone, i.e., the ratio of the response of the microphone to the sound pressure in a progressive wave into which it is introduced, the absorbent lined room shown in Fig. 8 is employed. This is roughly a 10-foot cube in external dimensions, and is lined internally with 6 inches of eel grass over which is 12 inches of cotton wadding suspended edgewise. The floor, which is treated in the same manner, is fitted with removable metal grids.

A loudspeaker situated at one end serves as a source of sound. The particle velocities at a series of points on the axis of the loudspeaker are determined from the deflections of a Rayleigh disk, measured from outside the cabinet by means of the usual light spot on the translucent scale shown. The corresponding sound pressures are deduced from the known relations in a progressive wave. The microphone is then substituted for the disk and the e.m.f. which it develops is measured at each of the same series of points. Such measurements are carried out at frequencies of from 10,000 cycles per second down to 300 cycles per second. At lower frequencies the residual reflections from the lagged walls are no longer negligible, with the result that the relation between sound pressure and particle velocity for a progressive wave is no longer valid. At such frequencies indirect methods are used for the determination of the field sensitivity.

The pressure sensitivity of the standard microphone, i.e., the ratio of the response of the microphone to the sound pressure actuating the diaphragm, is determined from Rayleigh disk measurements in stationary waves. The Rayleigh disk is suspended on the axis of a cylindrical pipe of cross-sectional diameter approximately equal to that of the microphone diaphragm, which closes one end of the pipe. Stationary waves are set up in the pipe by means of a loudspeaker opposite the open end. With the frequency adjusted so that the Rayleigh disk is at a velocity antinode, measurements are made of the deflection of the disk, from which the sound pressure at the diaphragm of the microphone is calculated, and of the microphone response. The pressure sensitivity has thus been determined over a range of frequencies from 3500 cycles per second to 62.5 cycles per second. The method is limited at the higher frequencies by the possibility of transverse vibrations occurring in the pipe, but at the lower frequencies the only limitation is the inconveniently long pipe required.

For frequencies below 62.5 cycles per second, therefore, an alternative method has been devised. This makes use of a type of pistonphone. A small piston driven by the moving coil of a loudspeaker unit, works into a small cavity of which one side is closed by the diaphragm of the microphone. A steel point attached to the underside of the piston bears upon the horizontal arm of a metal angle carried on a torsion strip. The vertical arm carries a mirror from which a spot of light is reflected to a scale. The deflections of the light spot when the piston vibrates enable the amplitude of motion of the piston to be measured, and consequently the sound pressures set up in the cavity to be calculated. With this instrument the pressure sensitivity of the standard microphone has been determined over the range of frequencies from 10 to 400 cycles per second.

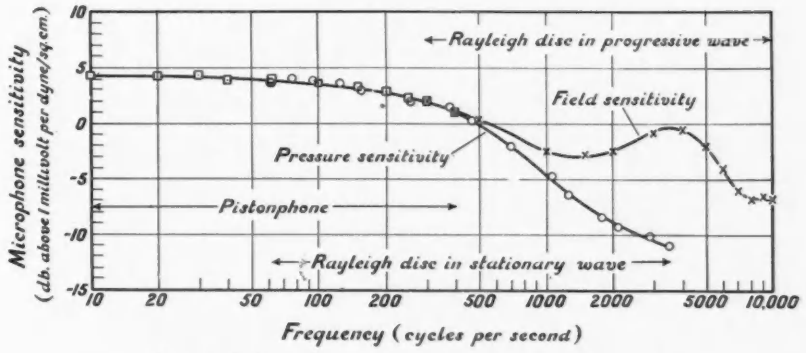

FIG. 9. Calibration curves of standard microphone.

The calibration curves of the standard microphone are shown in Fig. 9. The field sensitivity tends to be indistinguishable from the pressure sensitivity at frequencies below about 400 cycles per second. For a microphone of this size (about $7\frac{1}{2}$ cm in diameter) therefore a determination of the pressure-sensitivity at the low frequencies where the field sensitivity cannot be directly determined, suffices to complete the field calibration. Over the range 62.5 to 400 cycles per second, where measurements of the pressure-sensitivity have been made both by the Rayleigh disk and by the pistonphone, the agreement between the two methods is very satisfactory.

In the cases of "velocity" microphones, which respond to the pressure gradient, and of microphones which respond both to the sound pressure and to the pressure gradient, a different method must necessarily be used for the determination of the field sensitivity at low frequencies. The method employed utilizes a wooden pipe of about 2 feet square section and some 20 feet long, which is closed at one end. Stationary waves are set up in the pipe and the standard microphone, mounted in the closed end, determines the sound pressure at that point. The field sensitivity of the microphone under calibration is determined by calculating the response which would be obtained in a progressive wave, utilizing the relations between sound pressure and pressure gradient in stationary waves and progressive waves, respectively.

LOUDSPEAKERS AND OTHER ACOUSTICAL INSTRUMENTS

For measurements of the acoustical output of loudspeakers an open air site is employed on the roof of the Physics Building (Fig. 10). The loudspeaker is mounted on the parapet and faces outwards from the building. A calibrated microphone, carried on a projecting arm enables the sound pressure to be measured at distances up to about 8 feet. By rotating the arm the polar distribution of sound may be investigated. Suitably placed reflectors have been installed to throw reflected sound away from the region of the microphone. The loudspeaker is actuated by a heterodyne oscillator and the microphone response is recorded automatically, a type of circuit being used which eliminates disturbances from wind or extraneous noises. Harmonic distortion is also measured by analyzing the

FIG. 10. Open air site for loudspeaker testing.

sound output resulting from an electrical input of pure wave form.

Similar measurements are also undertaken on telephone receivers. In such cases the receiver is mounted on an "artificial ear" designed to impose upon it the same load as would be imposed by an average human ear, and the sound pressures developed within the cavity of the artificial ear are measured.

Miscellaneous tests undertaken include the measurement of frequency response curves of gramophone pick-ups and of electrical aids for the deaf, and the determination of the decay rates of tuning forks used by otologists in testing hearing. The miscellaneous researches include one on the velocity of sound in gases in tubes.[9]

Noise Measurements

For some years the measurement of noise and the investigation and analysis of noises of varied origin, with a view to their suppression, have formed a considerable proportion of the acoustic work of the Laboratory.[10–15] Loudness measurements have been made both by subjective methods, using instruments of the Barkhausen type, and by objective methods. The advantages and limitations of each type of instrument have been studied.

An instrument for the objective measurement of the equivalent loudness of noises of moderate intensity has been developed. This instrument, shown in Fig. 11, consists essentially of a condenser microphone, amplifier, rectifier and indicator. The gain of the amplifier may be varied in steps of 1 decibel, and electrical circuits are incorporated to correct for the variation with frequency of the sensitivity of the microphone, and also to reduce the response at low and high frequencies in accordance with the equal loudness curves at a level of about 70 phons. The instrument is designed to simulate still further the characteristics of the ear in that it deals substantially correctly with intermittent sounds, whether single impulses or successions of impulses at different rates of repetition. An account of the instrument will shortly be published.

Since 1929 a considerable amount of work has been carried out for the Aeronautical Research Committee on the various aspects of aircraft noise.[16] Measurements have been made on a number of aeroplanes in flight, and upon the noise from airscrews and engine exhausts under experimental conditions. In connection with the development of exhaust silencers an investigation has been carried out on model silencing conduits considered as acoustic filters.

The Laboratory has also carried out an investigation of train noises for a railway company, and is frequently consulted in connection with the suppression of noise in such situations as

Fig. 11. Portable acoustimeter for objective measurement of sound intensity and equivalent loudness.

[9] G. W. C. Kaye and G. G. Sherratt, "The Velocity of Sound in Gases in Tubes," Proc. Roy. Soc. **A141**, 123 (1933).

[10] G. W. C. Kaye, "The Measurement of Noise," Proc. Roy. Inst. **26**, 435 (1931).

[11] G. W. C. Kaye, "The Suppression of Noise," Eng. **134**, 314, 432 (1932).

[12] G. W. C. Kaye, "Some Scientific Aspects of Noise," *Anti-Noise League Exhibition Handbook*, 1935.

[13] G. W. C. Kaye, "Sound and Noise," Science Progress, January, 1936.

[14] A. H. Davis, "The Measurement of Noise by Means of a Tuning Fork," Nature **125**, 48 (1930).

[15] A. H. Davis, "The Measurement of Noise," Eng. **138**, 663 (1934).

[16] A. H. Davis, "Silencing Aircraft," Aeronautical Research Committee Reports and Memoranda No. 1542, 1933.

FIG. 12. Measurement of noise of motor vehicles.

aeroplane engine testing beds, printing works, transformer sub-stations, noisy offices and so on.

Recently a large program of work in connection with noise in the operation of mechanically propelled vehicles has been undertaken by the Laboratory for a Committee of the Ministry of Transport.[17] Objective measurements have been made of the loudness of the noise from a number of motor cycles, motor cars, and commercial motor vehicles, under various conditions, e.g., (a) when stationary with racing engine, and (b) when running at various steady and accelerating speeds both on the level, and when hill climbing. Fig. 12 shows measurements being made on a car with the objective noise meter. As a result of the investigation the Committee has submitted proposals to the Minister to forbid the offering for sale or for use on the public highway of motor vehicles which produce noise of loudness exceeding certain prescribed limits. In the case of private cars, for instance, the proposed limits are 90 phons at a distance 18 feet laterally from the center of the vehicle when running at a road speed of 30 m.p.h., and 95 phons at a distance of 18 feet behind the exhaust, when the vehicle is stationary and the engine is running at the speed at which it would give maximum power.

It may be added that these equivalent loudnesses are based on the British Standard scale of phons, which agrees with the American decibel scale in specifying as the standard reference tone a free air wave of 1000 cycles per second, with a zero of 0.0002 dyne per sq. cm, listening being conducted with both ears and with the observer facing the source of sound.

The Laboratory has also been called upon to carry out for the Ministry of Transport Committee an investigation of horns and other acoustical warning devices used on motor vehicles. The Committee proposes to make recommendations as to what regulations are desirable to control the characteristics of such devices with a view to abating any nuisance they may cause.

I am glad to acknowledge my indebtedness to Mr. N. Fleming, for his assistance with this article.

[17] Ministry of Transport, "First Interim Report of the Departmental Committee on noise in the operation of mechanically propelled vehicles," H. M. Stationery Office, 1935.

Standing Sound Waves in the Boehm Flute Measured by the Hot Wire Probe

ROBERT W. YOUNG* AND DONALD H. LOUGHRIDGE, *University of Washington*

(Received July 11, 1935)

IN comparison with other physical measurements the accurate observation of acoustic phenomena has been relatively more difficult. This has without doubt hindered in particular the study of musical instruments. Nevertheless, measurements of pressure variations in an organ pipe, for example, were long ago made by Töpler and Boltzmann.[1] The optical interference method, however, of these investigators could not easily be adapted to the study of other musical instruments.

By means of a very sensitive pressure measuring device attached to a long probe tube, Steinhausen[2] in 1914 was successful in observing the pressure amplitudes in the conical flute. More recently, along with the development of the hot wire microphone,[3,4] it was observed that when a heated filament is exposed to sound there results a constant or average decrease in electrical resistance which may be measured with a Wheatstone net. There are also periodic changes in resistances, at frequencies equal to and double that of the incident sound.

Employing the constant effect just mentioned, Richardson has measured the velocity amplitude in organ pipes and resonators.[5,6] His calibration curves for the hot wire in a direct-current draft of air showed the decrease in electrical resistance proportional to the square of the velocity for low velocities, just as reported by Tucker and Paris, and for high velocities the linear relationship observed by King.[7] He then assumed the change in resistance of the hot filament cooled by a harmonic draft to be the same as that produced by a d.c. velocity of magnitude equal to that of the maximum of the harmonic draft. Although this was the conclusion reached by Richards[8] in mounting hot grids on vibrating bodies, more recent experiments[9,10,11,12] indicate that the cooling is quite definitely a function of the frequency involved, as well as of the air velocity.

The present paper records the result of a series of measurements taken with a hot wire probe placed within the standing sound wave formed in the ordinary Boehm flute. This instrument lends itself conveniently to the making of such observations, due to its relatively simple geometry.

The various notes of the musical scale are marked according to the notation recommended by the Standardization Committee of the Acoustical Society.[13] The pitch standard $a_8=440$ cycles now used by American manufacturers has been assumed, although the actual recorded frequencies may deviate from the nominal equal tempered scale.

An optical bench was fitted with special clamps so that the flute with its various accessories might be supported as shown schematically in

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*Now with C. G. Conn, Ltd., Elkhart, Indiana.

[1] Töpler and Boltzmann, "Ueber eine neue optische Methode, die Schwingungen tönender Luftsaulen zu analysiren," Pogg. Ann. **141** (or **217**), 321 (1870).

[2] Steinhausen, "Untersuchung Stehender Luftschwingungen (Inbesondere in Flote und Orgelpfeife)," Ann. d. Physik **48**, 693 (1915).

[3] W. S. Tucker and E. T. Paris, "Selective Hot-Wire Microphone," Phil. Trans. Roy. Soc. **A221**, 389 (1921).

[4] W. S. Tucker, "Hot Wire Microphone Applications," Roy. Soc. Arts. J. **71**, 121 (1923).

[5] E. G. Richardson, "Amplitude of Sound Waves in Pipes," Proc. Roy. Soc. **A112**, 522 (1926).

[6] E. G. Richardson, "The Amplitude of Sound Waves in Resonators," Proc. Phys. Soc. **40**, 206 (1928).

[7] L. V. King, "On the Convection of Heat from Small Cylinders in a Stream of Fluid," Phil. Trans. Roy. Soc. **A214**, 373 (1914).

[8] R. C. Richards, "The Resistance of a Hot Wire in an Alternating Air Current," Phil. Mag. **45**, 926 (1923).

[9] A. von Hippel, "Die Theorie des Thermomikrophons," Ann. d. Physik **75**, 521 (1924).

[10] A. von Hippel, "Experimentelle Untersuchungen des Thermomikrophones," Ann. d. Physik **76**, 590 (1925).

[11] Goldbaum and Waetzmann, "Geschwingigkeitsmessungen mit erhitzten Drähten in stehenden Luftwellen," Zeits. f. Physik **54**, 179 (1929).

[12] Müller and Waetzmann, "Absolute Geschwindigkeitsmessungen mit Hitzdrähten in stehenden Schallwellen," Zeits. f. Physik **62**, 167 (1930).

[13] Report of the Committee on Standardization, J.A.S.A. **2**, 316 (1931).

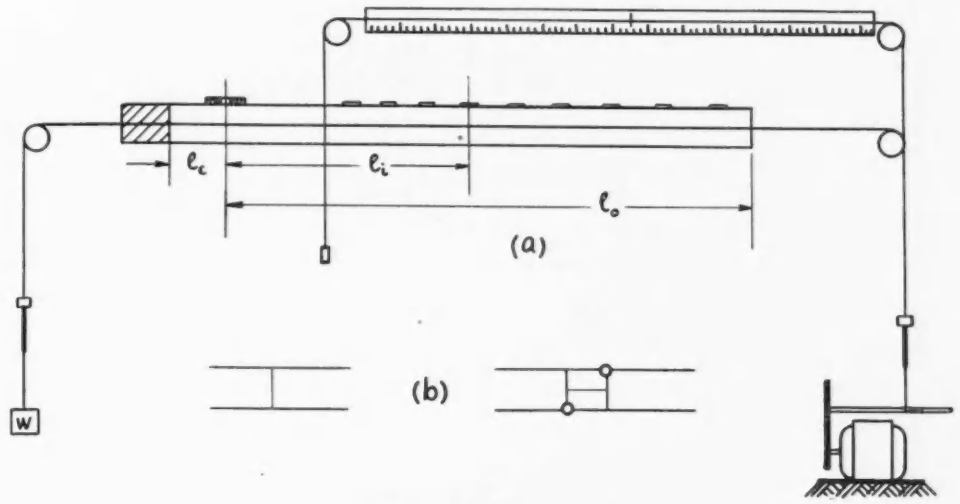

FIG. 1. Mechanical arrangement.

Fig. 1. The numbering system for the dimensions given in Table I is that given by Boehm,[14] with the addition of 8′ for the duplicate $g\sharp$ key. Since a hole always remains somewhat closed by its slightly lifted pad, the measurements entitled "lift" in the last column of Table I were so estimated that the effective lateral area of opening might be roughly approximated by multiplying the lift by the circumference of the hole in question.

The hot wire support consisted of two parallel copper wires (B & S No. 30) each approximately

TABLE I. *Flute dimensions.*

HOLE NUMBER	KEY NAME	HOLE RADIUS	HOLE POSITION	KEY LIFT
0			60.14	
1	$C\sharp$	0.783	55.80	0.12
2	D	.783	52.29	.12
3	$D\sharp$	.783	49.36	.15
4	E	.718	45.95	.15
5	F	.718	43.13	.12
6	$F\sharp$	.718	40.28	.18
7	G	.716	37.53	.13
8	$G\sharp$	.667	35.41	.13
8′	$G\sharp'$	.643	35.17	.12
9	A	.669	33.03	.12
10	$A\sharp$	.681	30.62	.12
11	B	.681	28.48	.11
12	C	.650	26.44	.16
13	$C\sharp$	.364	23.34	.10
14	D tr	.408	21.83	.25
15	$D\sharp$ tr	.408	20.18	.25
	Cork		−1.70	

All dimensions in centimeters.
All hole positions, l_i, have been figured from center of embouchure to center of hole in question.
Inside diameter of tube 1.90 cm, except tapered head joint which contracts to 1.77 cm at center of embouchure.
Approximate area of rectangular embouchure equals 1.13 sq. cm.

[14] Theobald Boehm, *The Flute and Flute Playing* (Dayton C. Miller, Cleveland, 1922), p. 79.

TABLE II. *Resistance changes along flute tube sounding c_8=260.4 cycles.*
R_h=6.16 ohms; i=0.157 amp.; R=5.39 ohms; S=2 ohms

POSITION	METER	ΔR_1	ΔR_0	POSITION	METER	ΔR_1	ΔR_0
3 cm	3.5	56	41.6	35 cm	29.3	226	9.7
4	3.1	52	31.6	36	35.8	255	10.0
5	3.3	54	24.8	37	42.4	284	10.5
6	3.0	50	24.5	38	48.8	308	10.9
7	5.3	76	23.5	39	54.3	332	11.2
8	10.0	116	19.0	40	59.7	354	11.8
9	18.0	170	12.4	41	64.2	372	12.0
10	32.0	238	9.5	42	69.0	390	12.4
11	44.0	288	10.8	43	72.1	402	12.8
12	48.3	306	10.5	44	75.0	414	13.0
13	49.3	310	9.4	45	75.0	414	13.5
14	49.0	308	7.7	46	80.0	434	13.9
15	44.0	288	5.7	47	82.3	444	14.3
16	44.6	292	5.9	48	84.5	452	14.7
17	44.4	290	6.2	49	85.0	454	15.0
18	41.5	280	6.2	50	86.1	460	15.2
19	36.0	256	6.2	51	87.2	464	15.7
20	30.8	234	6.8	52	88.2	468	16.0
21	25.6	208	7.0	53	89.0	470	16.2
22	19.5	178	6.8	54	89.3	472	16.5
23	14.5	148	7.2	55	89.2	472	16.6
24	9.5	108	7.4	56	89.8	474	16.8
25	5.6	78	7.5	57	89.2	472	16.9
26	2.7	48	7.8	58	89.5	473	17.1
27	1.0	18	7.8	59	90.0	476	17.3
28	0.7	10	8.2	60	88.7	470	15.7
29	1.5	28	8.4	60.5	54.5	332	11.1
30	3.6	56	8.5	61	17.2	166	9.5
31	8.0	100	8.6	62	2.2	38	8
32	11.6	128	8.8	62.5	1.4	28	5.9
33	17.2	166	9.1	63	0.8	12	5.9
34	23.5	200	9.5	off	0.5	0	

250 cm long attached to insulating yokes. These wires passed over fiber rollers and through a specially constructed insulating "cork." A constant speed motor and weight served to pull the wires in one direction or the other.

The actual probe was comprised of 16 mm of 0.001-inch platinum wire electrically welded to the copper wires in a jig, so that the filament lay horizontal and perpendicular to the axis of the flute. It was also possible to weld the probe wire so as to lie coaxial with the flute by the aid of short copper wires perpendicular to the main supporting wires. These short pieces were held in position by solder at one end and insulating wax at the other. The two methods are indicated in Fig. 1 at (b). Use of the transverse mounting only is described in the present work.

Utilization of long supporting wires under tension possesses the advantage over a rod support that the volume displaced by the wires remains the same regardless of the position of the hot filament, and the ratio of support cross section to that of flute is relatively small, being less than 0.0004. A pointer attached as shown at (a) in Fig. 1 moved before an adjustable scale, and thus made it possible to read directly the distance of the probe wire from the center of the embouchure. A nut on each of the lugs in the yokes by which the copper wires were attached facilitated the equalization of tension in the supports. The various tone holes could be closed by weights clipped to the regular key mechanism.

Considerable difficulty was experienced in finding some means of sounding mechanically the lower tones of the flute. The mechanism finally adopted independently allowed rotation about an axis, displacement perpendicular to this axis, and a means of adjusting symmetrically the width of the slit through which the air was forced. Each movement could be recorded by appropriate scales. The blowing mechanism as a whole was attached to the main supporting beam, and could be adjusted easily so that its axis of rotation coincided with the "outside" edge of the mouth hole. This edge thus divided the emitted air sheet.

It is perhaps of interest to note that for the formation of a given tone, the distance from slit to edge, or "height of mouth," and inclination of air jet are not nearly as critical as the width of slit. It was found very advantageous to close, by some means such as cotton or modeling clay, the space between the lower edge of the slit and the "inside" edge of the mouth hole. So far as the author is aware this need is not stressed in the usual vortex theory of the tone production. Considerable change of position of the blowing mechanism could be allowed after production of the tone, but certain positions seemed desirable to initiate the tone most easily.

Air ordinarily at about 22.5°C was supplied from a low pressure compressor system. A constant pressure air tank floating on the air line made it possible to maintain a fairly steady pressure of about 5 cm of water over a long period. The tank could also be used for estimating the rate of flow of air.

The Hot Wire Probe

In a standing sound wave there are both periodic temperature and velocity variations, the former attaining a maximum in the neighborhood of a vibration node and the latter at an antinode. The heated filament can be utilized for the measurement of either effect,[12, 15, 16] depending upon the temperature at which the wire is used.

For the present work heating currents of the order of 0.15 amp. were employed, resulting in wire temperatures, as computed from resistance changes, in the vicinity of 175°C. Goldbaum and Waetzmann[11] have shown that the temperature effect on the hot wire due to the standing wave is negligible at these temperatures, so that we are concerned here only with the velocity effect.

Unfortunately a complete theory of the action of a hot filament in a cooling harmonic draft is not available. Von Hippel[9] has considered the problem in a number of its aspects, but mathematical difficulties hinder the treatment of the velocity cooling. However, experimenters[3, 9, 12] seem to agree that a good approximation may be obtained by assuming that the change in resist-

[15] Friese and Waetzmann, "Relative Temperaturmessungen in stehenden Schallwellen," Zeits. f. Physik **29**, 110 (1924).

[16] Friese and Waetzmann, "Absolute Temperaturmessungen in stehenden Schallwellen," Zeits. f. Physik **34**, 131 (1925).

ance of a heated filament is proportional to the square of the cooling velocity. The sensitivity to periodic velocities decreases rapidly with increasing frequency, and is also subject to the natural or artificial convection current[10] passing over the wire. In the present work a further complication arises due to air turbulence.

As previously stated, when a hot wire is exposed to air vibration there are periodic resistance changes of frequency equal to and double that of the incident sound. Now, recalling that the change in resistance is observed to vary as the square of the velocity, on the assumption of a draft comprised of two vibrations of angular frequency ω and 2ω plus a unidirectional air current we write

$$(\Delta R)^{\frac{1}{2}}=f_0+f_1\cos\omega t+f_2\cos(2\omega t+\theta),$$

where ΔR is the entire change in the resistance of the hot filament. The function f_0 represents the contribution of the direct current of air, and may be a function of the turbulence and relative wire temperature. f_1 represents the sensitivity and amplitudes of the periodic contribution due to the vibration of angular frequency ω. This function will in general vary with the frequency concerned and wire temperature, as well as with position in the standing wave. f_2 has a similar significance for the double frequency vibration. θ is a phase angle introduced for generality. The above expression may of course be written

$$\begin{aligned}\Delta R=\Delta R_0&+\Delta R_1\cos(\omega t+\alpha_1)\\&+\Delta R_2\cos(2\omega t+\alpha_2)+\Delta R_3\cos(3\omega t+\theta)\\&+\Delta R_4\cos(4\omega t+2\theta),\end{aligned}$$

where $\Delta R_0=(f_0^2+f_1^2/2+f_2^2/2)$;

$$\Delta R_1=2f_1(f_0^2-f_0f_2\cos\theta+(f_2/2)^2)^{\frac{1}{2}};$$

$$\tan\alpha_1=(f_2\sin\theta)/(2f_0-f_2\cos\theta);$$

$$\Delta R_2=2((f_1/2)^4+2(f_1/2)^2f_0f_2\cos\theta+f_0^2f_2^2)^{\frac{1}{2}};$$

$$\tan\alpha_2=(f_0f_1\sin\theta)/((f_1/2)^2+f_0f_2\cos\theta);$$

$$\Delta R_3=f_1f_2;\ \Delta R_4=f_2^2/2.$$

This is very similar to the result obtained by Tucker and Paris.[3]

The term ΔR_0 is the so-called constant effect and can be measured on a Wheatstone bridge. Care should be observed here by noting that ΔR_1, although proportional to f_1, would not exist in the absence of f_0 and f_2. Further, when $f_0=0$ and $f_2=0$, the original expression becomes

$$\Delta R=f_1^2/2+(f_1^2/2)\cos 2\omega t.$$

This predicts that, when a harmonic draft of the single angular frequency ω cools the hot wire, the change in resistance consists of a "constant" part plus a periodic variation which is of frequency *twice* that of the incident sound. This condition may be explained physically by observing that the particle velocity attains a maximum in absolute value twice in every complete fundamental cycle, and the convection draft has in general a direction differing from that of the harmonic one.

Use has been made[17] of the experimental fact that a polarizing wind tends to increase the first harmonic term, ΔR_1. This is, of course, compatible with the above relationship, since in the absence of a double frequency air vibration ΔR_1 is directly proportional to f_0. On the other hand, experimenters using the standing wave formed in a Kundt's tube arrangement[11, 12] have found the double frequency resistance change greater than the single frequency term, and that the amplitude of the former varied as the square of the amplitude of the impressed periodic particle velocity. This too is consistent with the assumed relationship, since in the absence of an impressed double frequency vibration ΔR_2 becomes merely $f_1^2/2$.

In the present measurements in the Boehm flute, a polarizing wind is present of the order of 40 cm per sec. A rough measurement, taken by tuning for a double frequency effect in a manner to be described below, indicated that even in the complex tone necessarily present in the flute the reading due to ΔR_1 was at least ten times as large as that due to ΔR_2. Previous experiments[10, 11] have shown that the sensitivity of the hot wire to periodic air vibrations drops off enormously with increasing frequency. Thus the present observation means that f_2 is small compared with f_1, and we may consider $\Delta R_1 \cong 2f_1f_0$. Thus by measuring the periodic resistance change of frequency equal to that of the fundamental of the incident complex sound, we are measuring a quantity directly proportional to the amplitude of the first harmonic in the standing wave system.

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[17] Olson and Massa, *Applied Acoustics* (Blakiston), p. 136.

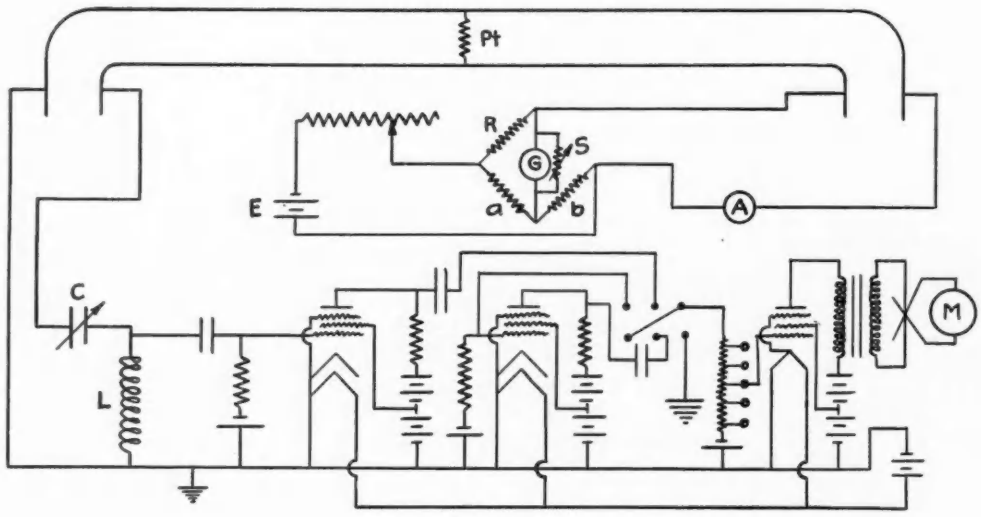

FIG. 2. Electrical connections.

ELECTRICAL MEASURING APPARATUS

In order to measure the "constant" effect mentioned above the hot wire with the supporting leads and ammeter were arranged to constitute one arm of a Wheatstone bridge. Since the necessary heating current for the hot wire passes through the bridge also, it was found convenient to use a Leeds and Northrup open dial type bridge immersed in oil. With a battery of 50 volts and series resistance, it was possible to keep the heating current approximately constant, despite changes of resistance in the hot wire. Fig. 2 shows the entire circuit.

The change in resistance of the hot probe when placed within the sounding flute was relatively large—sometimes as much as 20 percent. However, the variation in ΔR_0 along the tube was considerably less. Thus we find it convenient to set

$$\Delta R_0 = (R_h - \bar{R}) + \delta R,$$

where R_h is the resistance of the hot wire arm of the bridge when no cooling draft other than normal convection is present, and $\bar{R}$ is the average resistance of the wire over the standing wave system. Since we are interested in finding the variation in ΔR_0 as a function of position, we can do this by measuring δR, which differs from the former only by a constant.

As is well known, for a Wheatstone bridge slightly unbalanced[18] the galvanometer deflection can be taken as a measure of the amount off balance. To test this relation in the present case the probe wire was replaced by standard resistances. Galvanometer deflections were then observed corresponding to given resistance changes, the tests being conducted under different conditions of "heating" current, bridge setting, and galvanometer shunt S. Typical of these calibrations is that shown in Fig. 3. In this particular case $S=2$ ohms, the current $i=0.16$ amp. and the bridge was set for 4.63 ohms. A galvanometer reading of 25 corresponds to a balanced bridge. In this case as elsewhere

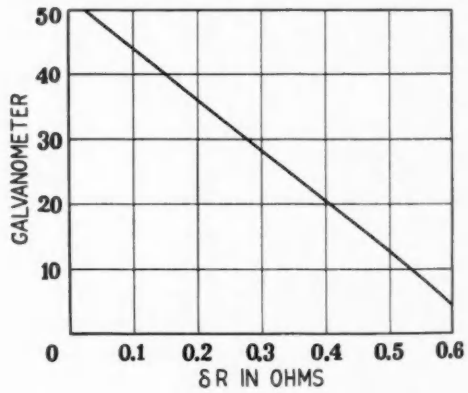

FIG. 3. Relation between galvanometer deflection and δR for 2-ohm shunt.

[18] Laws, *Electrical Measurements* (McGraw-Hill), p. 184.

throughout this work, curves pass exactly through experimental points unless otherwise noted.

Thus in order to study the relative variation in δR, or ΔR_0, along the standing wave we need only to plot galvanometer deflections. This practice will be followed in the data to be presented.

In order to observe the periodic resistance changes a tuned audiofrequency amplifier was employed. The circuit is shown in Fig. 2, and is essentially that described by Johnson and Neitzert[19] for the measurement of small alternating voltages. The input circuit was tuned by the variable condenser C. This condenser could, by the aid of a switching system and fixed condensers, be adjusted to all values up 1.1μf. The inductance L had an approximate value of 600 millihenries, and a resistance of 140 ohms.

The output transformer was matched to the 500-ohm heater of a General Radio vacuum thermocouple. A Rawson multimeter, which on its most sensitive range read 5 millivolts full scale, was utilized in measuring the thermocouple voltages.

This general arrangement including a tuned input circuit provided ample sensitivity. Also the sensitivity of the probe wire could be enormously increased by increasing the heating current; the passage of 0.3 ampere brought the wire but to a red glow. Half of this current was sufficient for measurements in most cases.

The over-all response of the amplifier system was obtained by the aid of a beat frequency oscillator, amplifier and potentiometer. Thus with a known alternating voltage of given frequency replacing the hot wire, the tuning circuit was adjusted for maximum response, and multimeter readings recorded as a function of input r.m.s. voltages. Fig. 4 is typical of the calibration curves obtained at the frequencies at which readings were taken later in the flute.

Since the impedance at resonance of the tuned circuit is inversely proportional to the capacity, the sensitivity of the electrical system increases with frequency, but this is more than counterbalanced by the decrease in sensitivity of the hot wire at higher frequencies. Unfortunately, at small amplitudes the accuracy of reading is much impaired, so that for measurements of this type some other output recording arrangement might be more suitable.

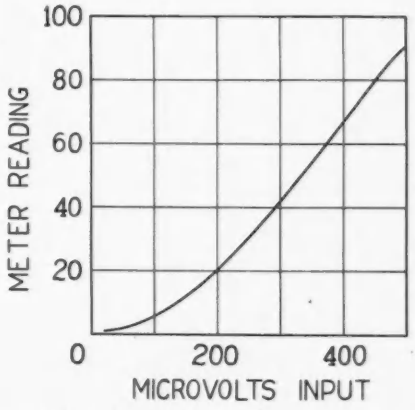

FIG. 4. Tuned amplifier response at 258 cycles.

RELATIVE VELOCITY MEASUREMENTS OF THE STANDING WAVE IN A FLUTE TUBE

The probe system previously described was connected in a smooth flute tube, entirely free of sockets or key mechanism. The head joint was then adjusted to make the length of tube from center of embouchure to end the same as that of the flute proper. With this tube sounding its fundamental, measurements of ΔR_0 and ΔR_1 were taken in the manner described above. A sample of such data is displayed in Fig. 5, and recorded in Table II.

Strictly speaking, the numbers recorded under ΔR_0 and ΔR_1 are only proportional to the actual resistance changes. It will be recalled that definitions were previously given so that $\Delta R_0 = (R_h - \bar{R}) + \delta R$. The numbers plotted as ΔR_0 are galvanometer readings, and are proportional to δR. Thus the true zero for ΔR_0 is considerably "below" that indicated in Fig. 5 by the amount $(R_h - \bar{R})$ so that the graph indicates only the variation at different positions along the tube. Numbers tabulated under ΔR_1 are voltages taken from the calibration. Blowing conditions varied considerably under different circumstances, so that it was found convenient to change the sensitivity of the hot wire by adjusting the heating current. Thus numerical comparison of the results described below is possible only in those

[19] Johnson and Neitzert, "The Measurement of Small Alternating Voltages at Audiofrequencies," Rev. Sci. Inst. 5, 196 (1934).

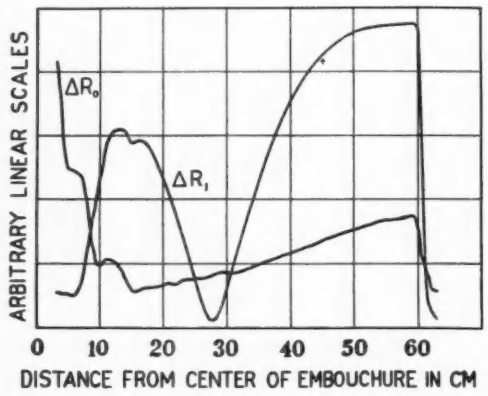

FIG. 5. Resistance changes along flute tube sounding $c_s = 260.4$ cycles.

curves where blowing conditions and wire sensitivity have been left unchanged.

Returning to Fig. 5 we shall consider some of its features. This curve, like the others to follow, has been passed through the experimental points taken each cm and has not been smoothed out. Because of the difficulty of reproducing blowing conditions, typical curves are shown rather than those plotted from numerical averages.

As observed in the organ pipe measurements,[5] the present curve shows a large constant resistance change near the mouth end of the tube. This reaches a minimum about 15 cm and then rises gradually as the end of the tube is approached at 60.1 cm, finally falling rapidly as the probe is carried out into the open air.

ΔR_1, on the other hand, assumes a rather low value near the embouchure, but rises and then falls to a node in the neighborhood of 28 cm. The word "node" will be used here to refer to a point of minimum vibration or velocity rather than an absolute zero. Beyond the node just observed, the curves rises smoothly to a maximum near the end of the tube, finally falling rapidly toward zero a few centimeters beyond the end of the tube.

Here we have experimental evidence of the vibration of the air beyond the open end of a sounding pipe, for which the "end correction" is usually added. This portion of the curve is of the "exponential" type, although with such a steep gradient it is difficult experimentally to prove an exponential character.

It is noteworthy to observe that the minima of ΔR_1 and ΔR_0 curves are far from coinciding. Such a discrepancy is characteristic of other curves to be shown below, and will be dealt with later.

By a readjustment of the blowing mechanism (principally enlarging the slit from which the air issued) it was possible to overblow the tube to sound the second harmonic. Fig. 6 shows the results of measurements taken with the tube being sounded in this manner.

If one disregards the first 13 cm shown in this curve, ΔR_1 approximates rather well a rectified cosine curve such as would be predicted by the usual simplified organ pipe theory. However, there is a slight rounding off of the curve at the junction of cosine and exponential parts near the end of the tube. It appears likely that this is due to a slight divergence in the flow lines, resulting in a reduction of velocity at the axis where the measurements were taken. This reduction may occur in either or both the particle velocity and what one might call the direct current velocity. On the other hand, a slight reduction of the sensitivity of the wire due to the nearby open air might also be sufficient to explain this rounding of the curve. We again note in this case the minimum in ΔR_0 about 16 cm.

By an increase in blowing pressure and further adjustment the flute tube was overblown to sound its third harmonic. While being sounded in this manner, the data of Fig. 7 were taken. We observe now that the curve is somewhat distorted for a greater distance from the embouchure than in the previous cases. This appears to indicate that a highly damped turbulence is present, which extends along the tube a distance somewhat dependent upon the original blowing pressure.

In the latter half of this curve we note a marked similarity in the undulation of ΔR_0 and ΔR_1. Let us suppose for the moment that in this part of the tube there exist only a constant wind of velocity v_0 and a particle velocity in a standing wave of angular frequency ω. Then assuming a standing wave of sinusoidal form, and amplitude factors B and C which include the sensitivity of the wire, we can replace the f functions used previously by $f_0 = Bv_0$ and $f_1 = C \cos(kx+\theta)$. Thus we have

$$\Delta R_0 = B^2 v_0^2 + \tfrac{1}{2} C^2 \cos^2 (kx+\theta)$$

or

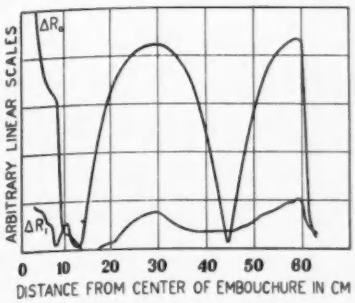

FIG. 6. Resistance changes along flute tube sounding $c_9 = 530$ cycles.

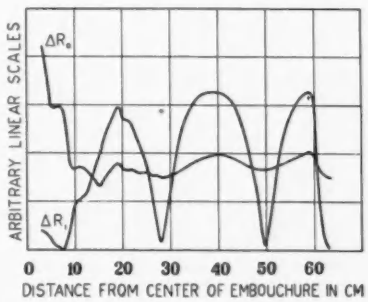

FIG. 7. Resistance changes along flute tube sounding $g_9 = 780$ cycles.

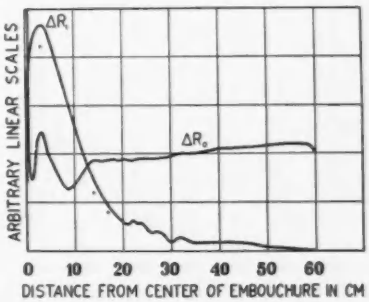

FIG. 8. Resistance changes along flute tube due to air forced through embouchure.

$$\Delta R_0 = \text{constant} + C^2 \cos (2kx + \theta),$$

and

$$\Delta R_1 = \text{constant} \cdot \cos (kx + \theta).$$

In the portion of the curve to which we have referred these assumed relations approximate the shapes actually observed, remembering of course that the hot wire does not give us phase relations.

This curve for ΔR_0 has also the prominent depression about 15 cm. Since the location of this minimum does not change with the frequency of the tone being sounded we conclude that this part of the curve must be influenced largely by a direct current of air of variable direction, and to turbulence which may follow the d.c. stream lines. Since air originally enters the flute transversely and not normal to its axis, it is reasonable to assume that a major portion of the air progresses down the tube for some distance in a cork screw fashion, largely along the wall. Since the probe wire passes only along the axis the value of ΔR_0 observed may be due principally to turbulence which has nearly died out before reaching the 15-cm mark. Beyond this position the through current of air may sufficiently rectify itself to a constant value so that the resistance changes due to periodic air vibrations may be observed.

On the other hand, within the first 15 cm the values of ΔR_1 appear unbelievably low. Since the sensitivity of the hot wire to periodic changes depends upon the rate at which heat is supplied after each cooling pulse has passed over, in all probability the turbulence which produces the abnormally high "constant" change in resistance carries off heat so rapidly as to decrease materially the sensitivity to periodic change.

In order to study the behavior of the direct air current, the regular blowing mechanism was rotated out of position and a simple tube fastened in its place. This tube was sealed to the embouchure with modeling clay at such an angle that air could be forced into the flute tube in a direction similar to that in which the blowing mechanism ordinarily emitted the air. Although a variation in ΔR_0 still existed no response was obtained through the tuned amplifier. However, by replacing the tuned circuit by another stage of amplification, data were obtained as shown in Fig. 8.

That cooling pulses of some kind are present is easily seen from the value of ΔR_1, remembering however, that the tuned circuit was removed here. It is interesting to note, also, that the values of ΔR_1 in the present case indicate a turbulence that drops off rapidly in much the same fashion as suggested by previous curves.

However, we are not justified in presuming that the d.c. velocity distribution of the present artificial case is the same as that existing when the flute is sounded. Nevertheless, the order of magnitude is probably correct, since the artificial draft was adjusted to produce about the same constant cooling near the end of the tube as when the flute was being sounded. Based on a uniform cross-sectional flow, air was being forced through the tube at about 43 cm/second.

Relative Velocity Measurements in the Standing Wave System of the Boehm Flute

It is now an easy step to the consideration of the standing wave system found in the Boehm

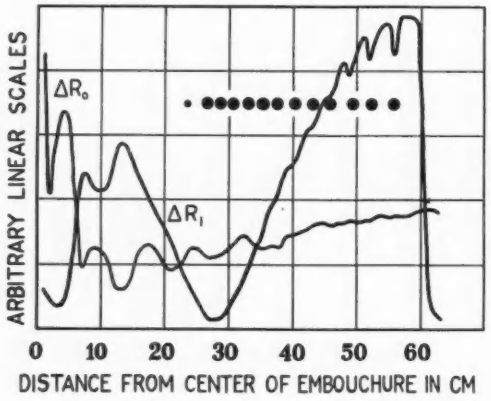

FIG. 9. Resistance changes along Boehm flute sounding $c_8 = 262.6$ cycles.

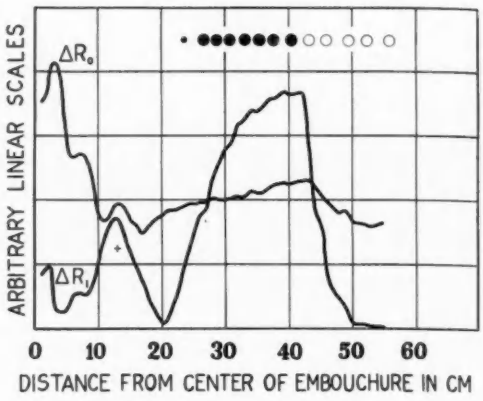

FIG. 10. Resistance changes along Boehm flute sounding $f_8 = 344.5$ cycles.

flute. Only one new factor has been added to the problem, namely, the presence of the closed tone holes. Their rather striking influence may be seen by reference to Fig. 9.

In the figures to be shown hereafter, in connection with the flute, the positions of the tone holes will be indicated by circles approximately to scale. A circle filled in shows that that particular hole is closed. Actually, of course, all tone holes are not in line, although shown thus for convenience. The presence of the duplicate $g\sharp$ key is indicated by overlapping circles, and a circle half filled in will mean this key is closed. Two small holes, for the d and $d\sharp$ trill, are not shown.

That the serrated portion of the curve is due to the tone hole sockets may be observed by comparing positions of the depressions and tone holes. Since there is effectively a sizeable expansion in bore at each closed tone hole, this naturally results in a velocity decrease as measured along the axis of the flute. Other characteristics are similar to those already pointed out in the case of the smooth tube.

By opening up holes along the flute we arrive at still another situation. In Fig. 10 we find the results of measurements taken with the flute "fingered" for f_8, that is, with holes all opened about one-third the length of the flute. It should be remarked that the letter f_8 is merely used to indicate the tone nominally expected when the f hole is open, but none open above it.

We see in Fig. 10 that the standing wave has, of course, become compressed, but we do not find the amplitude dropping off nearly as rapidly at the first open hole as it did for the fundamental at the end of the tube. Even though well "ventilated" by having all holes below the f hole open, the amplitude of ΔR_1 can be detected ten centimeters beyond the first opening reached by the sound. There also appears a tendency for this portion of the standing wave to continue with little diminution under those parts of the tube not directly open to the outer air. It may be possible, however, to consider this last part of the curve of exponential nature, but considerably less damped than at the open end.

Leaving all conditions the same as those under which the preceding observations were taken, all holes except No. 5 were closed and the measurements taken as shown in Fig. 11. Here we see that the "exponential" part of the preceding curve has been replaced by a steep slope and a sort of secondary standing wave pattern. The loading effect of this extra column of air can be estimated by noting the decrease in frequency of four cycles. Since, of course, the particle velocity is proportional to the frequency, the maximum amplitude of ΔR_1 in the present case is less than that of the preceding example.

Now we pass to the situation in which all holes, with the exception of three small ones nearest the embouchure and the $d\sharp$ hole, are open. The results for this case are given in Fig. 12. Here again we find the "exponential" portion of ΔR_1 under the open tone holes. The nodal position has now moved up to 12 cm, but the minimum in ΔR_0 still hangs close to 15 cm,

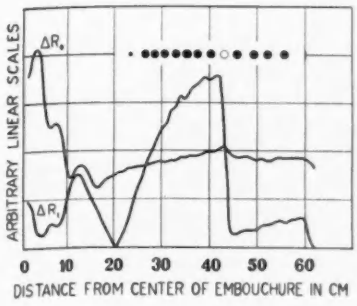

FIG. 11. Resistance changes along Boehm flute sounding $f_8 = 340.5$ cycles.

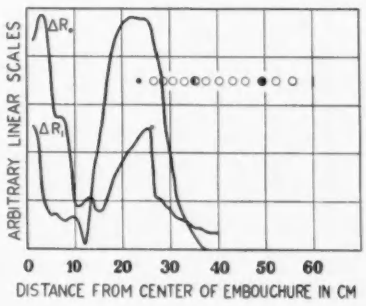

FIG. 12. Resistance changes along Boehm flute sounding $c_9 = 526$ cycles.

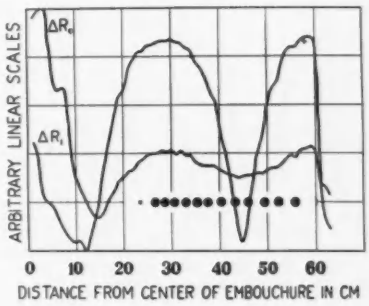

FIG. 13. Resistance changes along Boehm flute overblown to $c_9 = 524$ cycles.

bearing out the observation made some time previously. Beyond the first open hole ΔR_0 decreases, but not by an amount comparable to its total true value. This tends to show that little of the air originally forced in escapes through the side holes.

Since c_9 may also be sounded as the second harmonic of the entirely closed tube, it is interesting to compare the two different standing waves yielding about the same frequency. Fig. 13 yields the information in the overblown closed flute. The graph here is very similar to that of Fig. 6, excepting of course, the notches resulting from the tone hole cavities. Both Figs. 12 and 13 show a node very nearly in the same position, but the antinode positions differ considerably.

Passing now to the second octave of the flute's range, we consider the note f_9 sounded with the normal fingering. Measurements for this note are given in Fig. 14. It should be remarked that the frequencies for this and higher notes were estimated by comparison with forks about an octave lower, so that the frequencies recorded for these higher notes do not have the accuracy of frequencies given previously.

As seen on Fig. 14 the second node is clearly defined in the neighborhood of 33 cm, but the position of the first node is somewhat in doubt. There is, of course, the minimum very near 9 cm, but unfortunately this point is in a region where previous curves likewise indicated certain irregularity. The increase in blowing pressure necessary to cause the flute to overblow would, of course, tend to increase the distortion.

Fig. 15 shows the data taken with the flute overblown to sound f_9 with but one hole open. The graph shows a surprising amplitude of ΔR_1 near the end of the flute. A similar situation existed for the same arrangement of tone holes (see Fig. 11) and the sound an octave lower, but it was by no means the extreme shown in the present case. Apparently the ease with which the wave motion is propagated past the opening at 43 cm increases with frequency. This is, of course, a case with only one hole open, and a condition which is avoided in the playing of wind instruments. It illustrates well, however, the need of having several consecutive open holes for the purpose of avoiding the further complication of the secondary standing wave shown here.

Since the data on Figs. 14 and 15 were taken under the same conditions of wire sensitivity and setting of blowing mechanism, we can compare the two quantitatively. Frequency and maximum amplitude dropped in the second case in much the same way as in the previous situation an octave lower. This time, however, there is an unmistakable shift in the node at 33 cm of nearly a centimeter, although a faint indication of the same thing is noticeable upon comparison of Figs. 10 and 11 in the neighborhood of 20 cm. This shift might naturally be expected to accompany the decrease in frequency following the closing of a number of holes. Also the "quarter-wave-length" section from 48.5 cm to the end of the tube at 60.1 cm approximates half the "half-wave-length" portion from 9 cm to 33.5 cm. In view of the possible distortion mentioned above, one might well be justified in extrapolating the curve to find a node about 10.5 cm, in which case the comparison just made becomes much better.

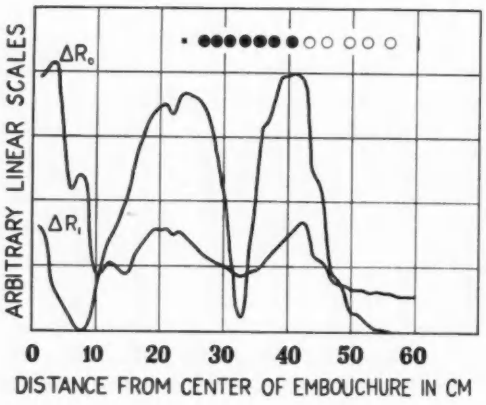

FIG. 14. Resistance changes along Boehm flute sounding $f_9 = 688$ cycles.

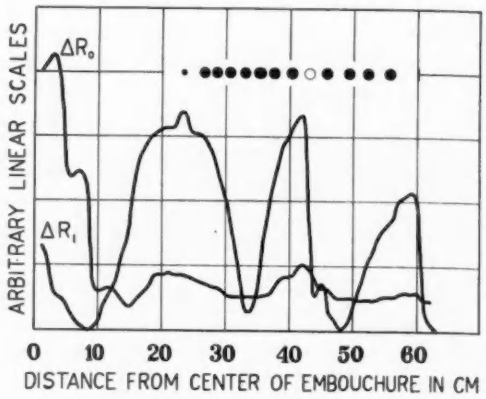

FIG. 15. Resistance changes along Boehm flute sounding $f_9 = 675$ cycles.

That a measured minimum in ΔR_1 exists near the 9 cm position there need be no doubt, since the experimental fact can be reproduced quite easily. It is possible that turbulence of some kind does tend to shift the node in this way, as it is well known[20] that a large "mouth end correction" must be added to the geometrical length of organ pipes and flutes in order to apply the simple theory usually assumed. As turbulence is always apt to exist near the mouth end of pipes in which the tone is produced by blowing toward an edge, it may well be that the need of this end correction follows as a result of the vortex motion present. However, since this same turbulence might merely affect the probe readings without an actual shift of the vibration node, it is regrettable that a lack of more complete information on the action of the hot wire under such circumstances forbids a more positive interpretation at the present.

As an example of the standing wave system for a note in the third octave of the flute's range, we show Fig. 16. Sounded by the aid of the usual fingering, this tone shows a standing wave much as would be expected on the basis of the preceding measurements: namely, a large value of ΔR_0 near the mouth end falling to a minimum about 15 cm, and an irregular variation of ΔR_1 changing to a sinusoidal type curve and finally falling off in an exponential fashion. Within the first 15 cm there is also the frequently observed correlation between the maxima of ΔR_0 and the minima of ΔR_1.

CONCLUSIONS

In common with probe measurements in many branches of physics, it is impossible to insert into a vibrating air column a probe that will measure the condition existing previous to the insertion of the foreign body. However, it appears that by the assistance of a hot filament supported by some method such as that described above, considerable knowledge may be gained concerning the standing wave system formed within musical instruments. The measurement of the periodic resistance change of a hot wire within a vibrating air body offers an advantage in the observation of the wave processes over the measurement of the constant resistance change, although the latter too can contribute to the

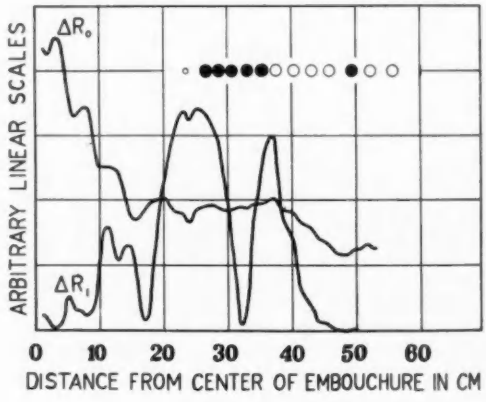

FIG. 16. Resistance changes along Boehm flute sounding $d_{10} = 1100$ cycles.

picture of the actual conditions under which musical instruments are played.

The present work demonstrates and measures the existence of a condition which has long been recognized but which has defied theoretical treatment—namely, the air vibration beyond the open end of an unflanged pipe. The mathematical description of musical instruments with side holes is indeed a difficult one, even with the general theory and its simplifications introduced recently by Irons.[20, 21] It is a well accepted fact in practice that, in musical instruments with side holes, it is most desirable for the proper production of a given tone to have a series of open holes below the one which largely determines the frequency concerned. The present work indicates that the standing wave has a noticeable amplitude extending beyond about three open tone holes, which happens to be the same number recommended as desirable by Boehm,[22] and the effect of omitting these "resonance" holes is also shown. Irons has successfully applied his theory to a musical instrument having but one side hole; he has encountered difficulties in the consideration of two openings. Actual musical instruments of the wood-wind group frequently have more than twenty side holes.

The present purely experimental work demonstrates the possible approximation that in the presence of open tone holes the amplitude of the standing air vibration is limited in an approximately exponential fashion, the exponential coefficient depending upon number, size, and location of openings. It is hoped that the experiment will offer further means for the extension of the theory of musical instruments.

[20] E. J. Irons, "Fingering of Wind Instruments," Phil. Mag. **10**, 16 (1930).

[21] E. J. Irons, "On the Fingering of Conical Wind Instruments," Phil. Mag. **11**, 535 (1931).

[22] T. Boehm, *On the Flute*, edited by W. S. Broadwood (Rudall, Carte & Co., 1882), p. 21.

Vortex Noise from Rotating Cylindrical Rods

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A series of round rods of the same diameter were rotated individually about the midpoint of each rod. Vortices are shed from the rods when in motion, giving rise to the emission of sound. With the rotating system placed in the open air, the distribution of sound in space, the acoustical power output, and the spectral distribution have been studied. The frequency of emission of vortices from any point on the rod is shown to be given by the formula of von Kármán. From the spectrum, estimates are made of the distribution of acoustical power along the rod, the amount of air concerned in sound production, the "equivalent size" of the vortices, and the acoustical energy content for each vortex.

INTRODUCTION

THE flow of air past stretched wires has been known for many centuries to produce musical tones, and this phenomenon forms the basis of the ancient Aeolian harp. This instrument consisted of a number of taut wires of graduated lengths fastened to a frame. The device was placed in a location subject to strong draughts and a succession of pure tones resulted as the air speed changed in magnitude.

The first investigator to examine this phenomenon was Strouhal.[1] He attached vertical wires to a drum in such a way that they could be rotated about an axis parallel to their length. He found the generated frequencies to be independent of the material, length, and tension of the wire, and that they depended only on the speed of rotation and the diameter of the wire. The generated frequency proved to be

$$f=0.185\,V/d \text{ cycle},$$

where V is the relative velocity of the wire and the medium and d is the wire diameter.

Lord Rayleigh showed that the wire itself partook of the vibration to a slight extent, and that the oscillation took place at right angles to the wind.[2] When the wire was tuned to the frequency f, the sound was greatly reinforced and the vibration perpendicular to the wind increased. This condition obtains in the Aeolian harp.

Von Kármán and Rubach were the first to suggest a rational explanation of the singing of wires.[3] Their investigation of the stability of a row of vortices shed from an obstacle in an air stream showed that the row could exist only under certain conditions, *viz*, if the vortices were released alternately from opposite sides of the obstacle at such a rate that the frequency

$$f=0.194\,V/d \text{ cycle},$$

where the symbols have the same significance as before. The alternation of release accounts for the vibration of the wires perpendicular to the air stream and the coefficient 0.194 is in good agreement with Strouhal's experimental value 0.185.

Relf reexamined the whole question experimentally with respect to vortices.[4] He found that the frequency of vortex formation behind wires immersed in liquids was given by the von Kármán formula; he repeated the work of Strouhal and obtained practically the same coefficient in the frequency formula.

Richardson tried obstacles of different shaped cross sections in liquids of widely different viscosities and obtained substantially the same results.[5]

Fage and Johansen investigating the air flow behind flat plates, found the coefficient to be close to 0.15 provided that the quantity d was taken as the width of the plate projected perpendicularly across the air stream.[6]

* Associate Physicist.

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[1] Strouhal, Ann. d. Physik **5**, 216 (1878).

[2] Rayleigh, Phil. Mag. **29**, 433 (1915).

[3] von Kármán and Rubach, "Mechanism of Liquid and Air Resistance," Physik. Zeits. **13**, 49 (1912).

[4] Relf, "Aeolian Tones in Air and Water," Phil. Mag. **42**, 173 (1921).

[5] Richardson, "Aeolian Tones," Proc. Phys. Soc. **36**, 153 (1924).

[6] Fage and Johansen, "On the Flow of Air behind an Inclined Flat Plate of Infinite Span," British Aero. Res. Committee, Rep. No. 1104, 1927.

Tyler, using a vibration galvanometer, studied the frequency of vortex release behind cylinders, plates and airfoils in air, and in liquids.[7] His results with cylinders showed nothing new. For plates he obtained a coefficient of 0.158 and for airfoils, 0.150. This value of the coefficient holds for an airfoil only when it is placed at large angles to the air stream, say from 30° to 90°. Below 30° the coefficient varies widely.

The von Kármán frequency formula is thus well established for nearly all shapes of obstacles, with the exception of airfoils at normal angles of attack.

In the case of a full-sized propeller blade, an appreciable fraction of the total acoustical power output arises from the release of vortices. Since a different frequency is emitted for every radius, the resulting sound spectrum is continuous from zero frequency to some definite frequency at which the spectrum stops.

In order to obtain information of this type of sound under simpler conditions than obtained with airfoils, a study was made of the sound emission from cylindrical rods rotated about their midpoint.

Experimental Arrangement

One-half inch diameter cylindrical rods were used. The ends were cut off squarely. The rods were divided in the middle for insertion into a metal hub; from tip to tip the lengths of the rods were 12, 18, and 24 inches.

The rods were mounted in a vertical plane at the end of a 36-inch horizontal shaft; they were driven by a $\frac{1}{4}$-horsepower electric motor. The entire system was mounted on a wooden frame at a height of about 6 feet from the ground. With this arrangement, tip speeds up to 50–60 meters per second could be attained. The frame was placed outdoors during the course of the experiments to eliminate reflection from walls.

The sound was picked up by an electrodynamic microphone and amplified in the conventional manner. Where necessary, analyses were made with the N. A. C. A. analyzer.[8] The interpretation of such analyses where the spectrum is perfectly continuous is to some extent uncertain; i.e., it is not definitely known whether a large amplitude at a certain frequency on the record is actually the result of a large sound pressure at that frequency or is the result of regularity of emission at that frequency by comparison with adjacent frequencies. Until further information is available, it will be assumed in this paper that the records give true sound pressures.

Distribution of Sound About the Rods

Observations of sound pressure about the 18-inch rod at a distance of 16 feet when rotating at 2800 revolutions per minute gave the distribution shown in Fig. 1. This type of polar diagram would be expected if the vortices on the two faces of the rotation disk were of opposite sign, in accordance with the von Kármán conception of vortex emission; destructive interference would have its maximum effect in the plane of rotation.

All rotating systems show this distribution for vortex noise provided that no obstacle of appreciable size is close enough to disturb the diagram. If, for example, a large driving motor is present directly behind the rotating member, the diagram will retain the correct shape in front but will be distorted toward the rear.

Acoustical Power Output

If the distribution of sound pressure about the rods is assumed to be a perfect figure of eight, it becomes possible to measure the power output in sound from pressure measurements at one angular position, say along the axis of rotation. The total power output will then be one-half of that computed on the basis of a circular distribution, since the area of a figure of eight is one-half that of the circumscribed circle.

Pressure measurements were taken along the axis of rotation at distances of 1, 2, 4, 8, 16, 32, and 64 feet from the 18-inch rod rotating 3050 times a minute. The product of sound pressure and distance should be a constant if the inverse-square law is obeyed; this condition must be satisfied in order that the equations for power transmission may apply. Table I shows the behavior of this product.

[7] Tyler, "Vortex Formation behind Obstacles of Various Sections," Phil. Mag. **11**, 849 (1931).

[8] Theodorsen, "A New Principle of Sound Frequency Analysis," National Advisory Committee for Aeronautics, Rep. No. 395, 1931.

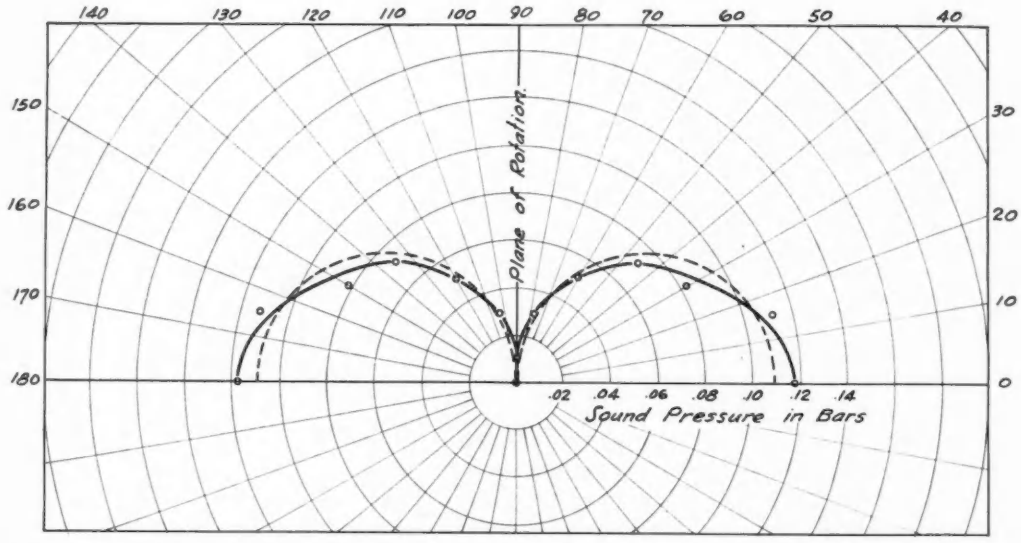

FIG. 1. Polar distribution of sound pressure about rotating rod. Solid curve, observed; dotted curve, computed.

The inverse-square law is approximately obeyed between 4 and 64 feet, inclusive. The mean value of the product in this range is 2.179 bars-feet, or 66.4 bars-centimeter. A circular distribution would yield a value of 66 microwatts for the power output. The true power in sound is therefore 33 microwatts at the speed used, 3050 r.p.m.

Observations were taken of the sound pressure at a distance of 6 feet in order to observe the effect of varying the rotational speed. The results are shown in Table II.

TABLE I.

DISTANCE (ft.)	SOUND PRESSURE, (bars)	PRODUCT, (bars·ft.)
1	1.275	1.275
2	0.863	1.725
4	.488	1.95
8	.242	1.935
16	.120	1.92
32	.0825	2.64
64	.0375	2.40

TABLE II.

SPEED (r.p.m.)	SOUND PRESSURE (bars)	ACOUS. OUTPUT (microwatts)	SPEED (r.p.m.)	SOUND PRESSURE (bars)	ACOUS. OUTPUT (microwatts)
980	0.0075	0.0135	2200	0.158	6.00
1200	.030	.22	2405	.203	10.00
1400	.045	.48	2620	.255	15.6
1600	.0675	1.09	2830	.316	24.0
1800	.0825	1.63	3035	.360	31.2
2000	.120	3.46			

A plot of this relation shows that the power output in sound is proportional to the $5\frac{1}{2}$ power of the rotational speed.

Relative observations were also made of the sound pressure from the 12-inch and 24-inch rods. The increase in sound pressure with tip speed is shown in Fig. 2. The relation is

$$\text{Power output in sound} = (\text{constant}) l (\text{tip speed})^{5.5},$$

where l is the length of the rod. Thus the controlling factor that determines the amount of sound is shown to be not the rotation speed but the tip speed.

SPECTRUM OF THE NOISE

Spectrograms of the vortex sound obtained with the N. A. C. A. sound analyzer are shown in Fig. 3. These records describe the frequency distribution of the sound when the 24-inch rod was rotated at four rotational speeds: 1080; 1250; 1750; and 2250 r.p.m. The microphone was placed 16 feet from the hub on the axis of rotation. The spectra all show one maximum which moves out to higher frequencies as the rotational speed increases.

Strouhal[1] who worked with fine wires stretched around the circumference of a drum in such a way that their motion was perpendicular to

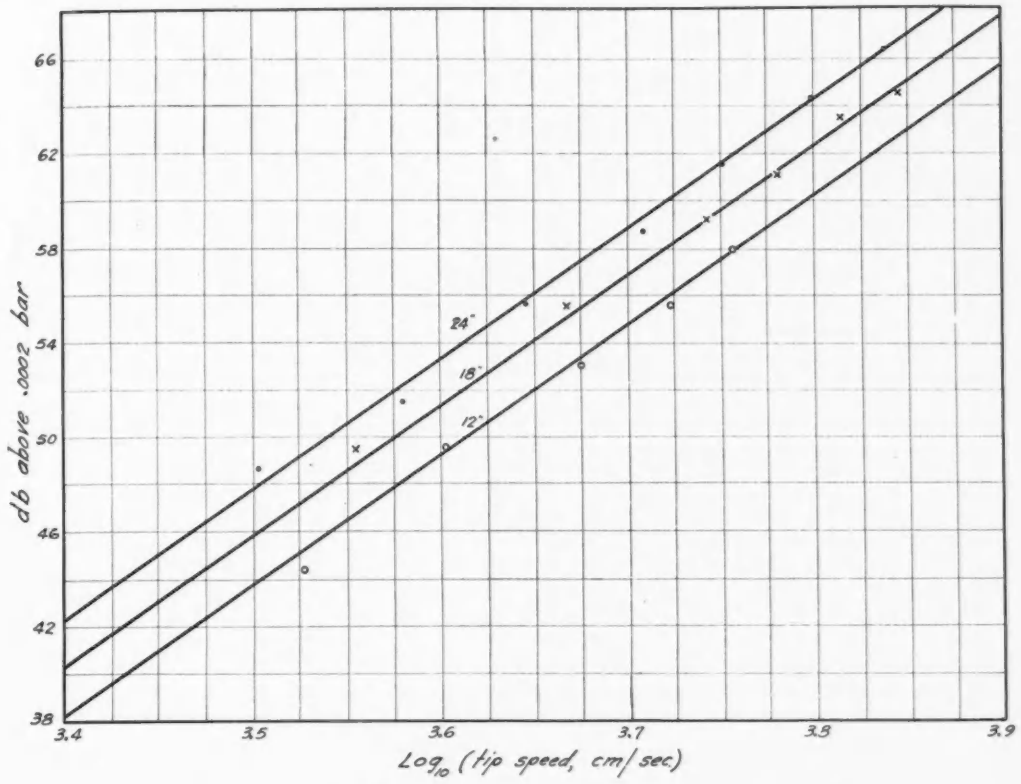

FIG. 2. Variation of sound pressure with tip speed for three rods.

their length, found that the generated frequency was given by

$$n = 0.185\, V/d \text{ cycle},$$

where V is the relative velocity of wire and medium and d is the diameter of the wire.

A rotating rod should therefore generate all frequencies from zero to a maximum determined by the value of V at the tip (neglecting the effect of the square ends). The maximum frequencies to be expected in the case of the records in Fig. 3 are listed in Table III.

A line has been drawn across the records at n_{max}. In all cases the line is noticeably past the maximum amplitude; it is evident, nevertheless, that the formula gives the frequencies generated with fair accuracy.

TABLE III.

ROTATIONAL SPEED (r.p.m.)	V (cm/sec.)	n_{max} (cycles)
1080	3460	532
1250	4000	615
1750	5620	863
2250	7210	1105

The form of the wave from any given point on the rod cannot be very complex for the spectra never extend beyond the frequency $2n_{max}$. The most that can be said is that the wave contains an appreciable amount of second harmonic.

As a further check upon the frequency formulas, spectra were recorded of the sound from all three rods, rotating at the same tip speed (5040 centimeters per second). The expected maximum frequency should be the same in all cases, 780 cycles. Fig. 4 shows the three records obtained; the spectra are seen to occupy the same frequency region. One feature of particular interest in these records is the growth of the isolated frequency at the left of the record. This frequency is absent in the case of the most slowly revolving rod (24 inches). The 18-inch rod shows a trace of 70 cycles, twice the revolution speed; whereas the 12-inch and most rapidly rotating rod exhibits a very pronounced component at 100 cycles, also twice the revolution

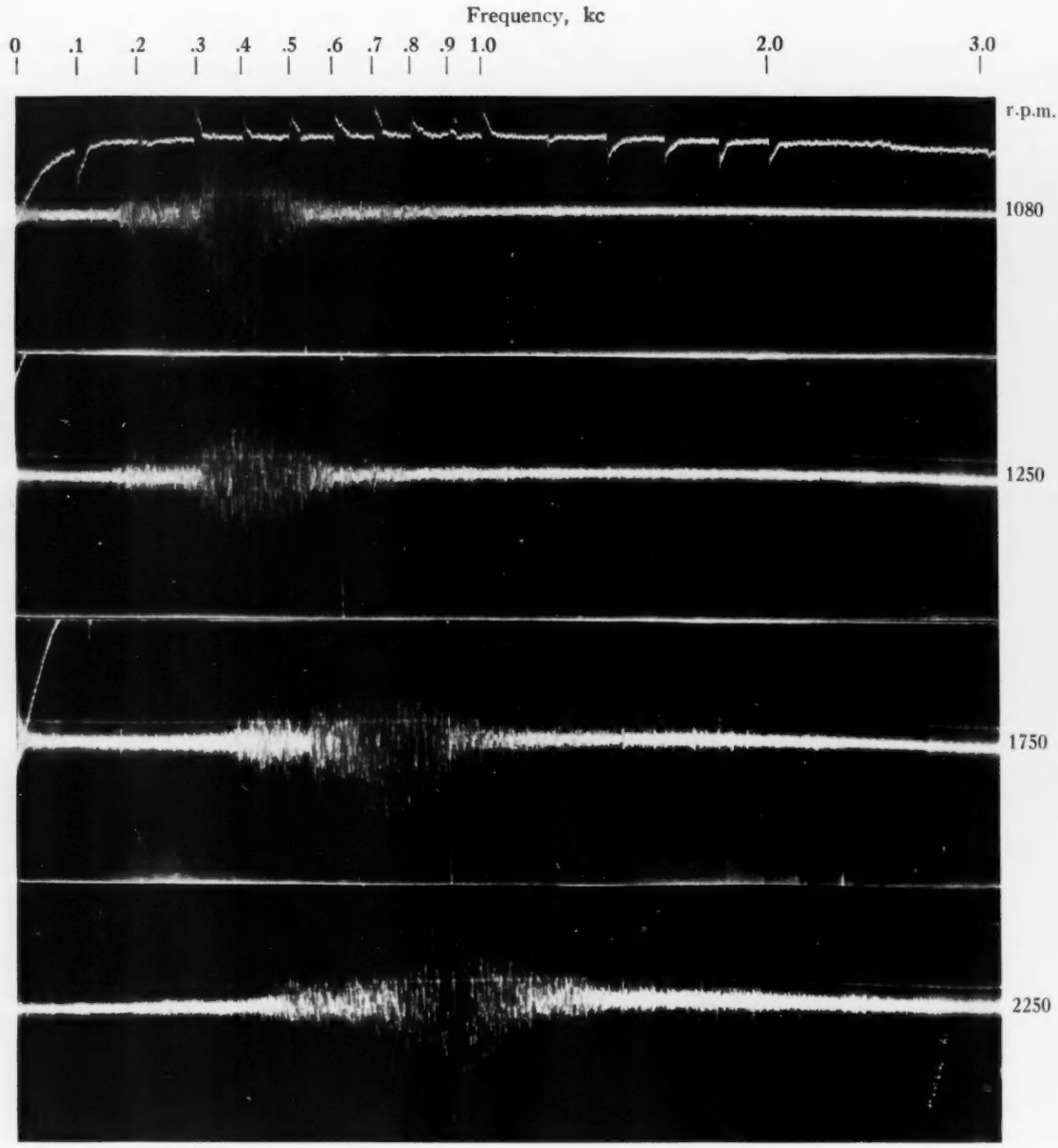

FIG. 3. Sound spectra from rotating 24″ rod at four speeds of rotation. Vertical lines indicate computed frequencies for tips.

speed. The amplitude of this pure note evidently depends upon the rotational and not the tip speed. In both cases where the note was present, it was inaudible.

The spectra observed with rods of $\frac{1}{4}$-inch instead of $\frac{1}{2}$-inch diameters were the same shape, but the frequencies were twice as great, in agreement with the Strouhal formula.

Specially designed rods were constructed with a small cylindrical segment at only one portion of the rod, the remainder being thinner and of a shape to reduce drag. It was hoped by such means to obtain a more or less isolated peak in the spectrum corresponding to the emission of a very narrow band of frequencies from the cylindrical portion. This peak would permit a more

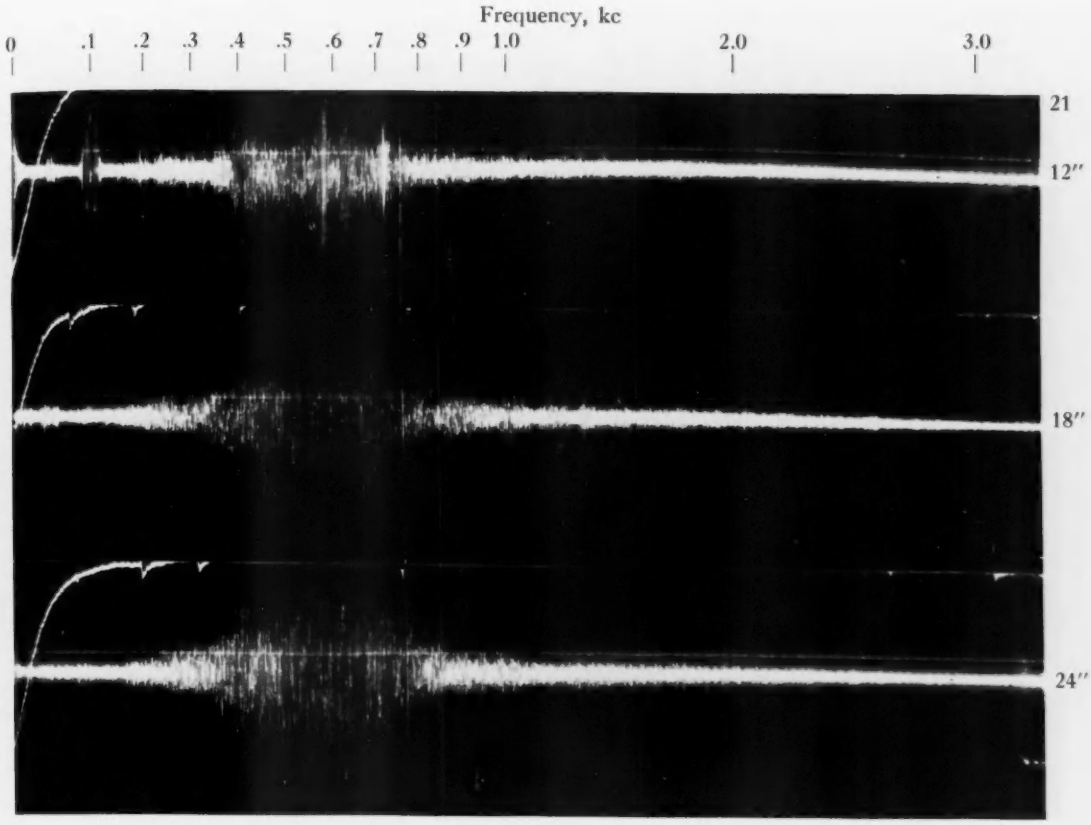

FIG. 4. Sound spectra from three rods rotating at same tip speed. Vertical line indicates computed frequency for tips.

exact verification of the frequency formulas. All such attempts were unsuccessful. In no case was a large enough peak found near the expected position to ascribe it, without question, to the action of the cylinder. Until further information is available, the formulas may, therefore, be taken as correct.

PRESSURE FIELD ABOUT THE ROD

If the Strouhal-von Kármán formula is taken as correct for every point on the rod, it becomes evident that the sound spectrum, considered now as a function of distance along the rod, must be closely associated with the distribution of pressure along the rod when in motion.

Accordingly, the pressure fields surrounding the rods were studied with a microphone a short distance from the rods. In order to diminish the effective area of the microphone and obtain more exact point-by-point pressure readings, a cap was fitted over the microphone completely enclosing it and a short tube was fitted into the cap. The open end of the tube was then used as the pressure probe to replace the microphone. The open end of the tube was kept at a fixed distance from the plane of rotation.

The variation of the pressure field at any point for any rod is shown in Fig. 5. The magnitude of the pressure variation is proportional to the 1.36 power of the rotational speed.

The distributions of pressure along the 12-inch rod for four speeds are shown in Fig. 6. In ascending order the speeds are: 1500; 2250; 2700; and 3500 r.p.m. The maximum pressure at the highest speed was 11,150 bars. The position of maximum pressure gradually shifts away from the tip as the speed is raised.

Similar data taken with the other two rods show the existence of the following relation:

$$\text{pressure at } x = \text{constant } l^{0.76} n^{1.36} x\varphi(x/l),$$

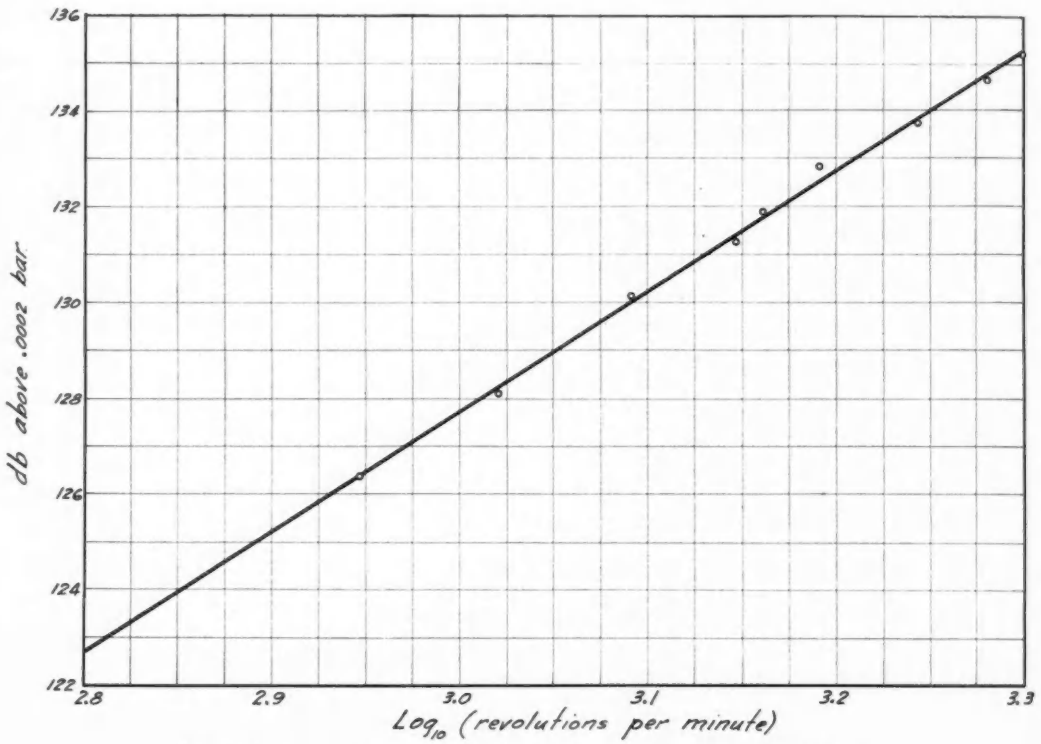

FIG. 5. Increase of pressure field about rod with speed of rotation.

where x is linear distance measured from the hub. The function $\varphi(x/l)$ may be of the general form $e^{-k(x/l)^2}$. The exact form of $\varphi(x/l)$ is indeterminate to the extent that our knowledge of the accuracy of the frequency formula is incomplete; i.e., the length of rod is a rather indefinite quantity with respect to the extent of the pressure field.

A comparison of the curves of Fig. 6 with the spectra of Figs. 3 and 4 shows a qualitative similarity between the pressure field and sound pressure, considered as functions of x.

DISTRIBUTION OF ACOUSTICAL POWER ALONG ROD

It is known from a previous section that the 18-inch rod when rotating about 2800 times a minute emits about 24 microwatts in sound. It should be possible to effect a division of this power among the frequency bands, and so distribute the power as a function of distance along the rod.

A difficulty immediately arises owing to the fact that the spectrum appears continuous. Any section of the rod large enough to include a vortex would therefore appear to be the source of an infinite amount of acoustical power. The difficulty is only apparent, however, as may be seen from the following argument. The release of a single vortex is accompanied by a single pressure disturbance. Both phenomena are independent of happenings immediately adjacent to the released vortex; the result is that the emitted sound is not composed of regular wave trains but of isolated pressure impulses. From instant to instant the configuration of vortices on the rod changes in an irregular fashion, with the result that over a period of time required for the analyzer to pass its own band width a sound pressure appears at all possible frequencies in the band. Thus more sound appears to have been emitted than was actually the case.

In order to reduce the spectrum actually obtained to an equivalent spectrum that would be obtained if the emission took place in wave trains, it is necessary to divide the recorded spectrum into frequency bands of such a width that the vectorial sum of the pressures found in the middle of the bands will equal the total pressure measured by a microphone. The band

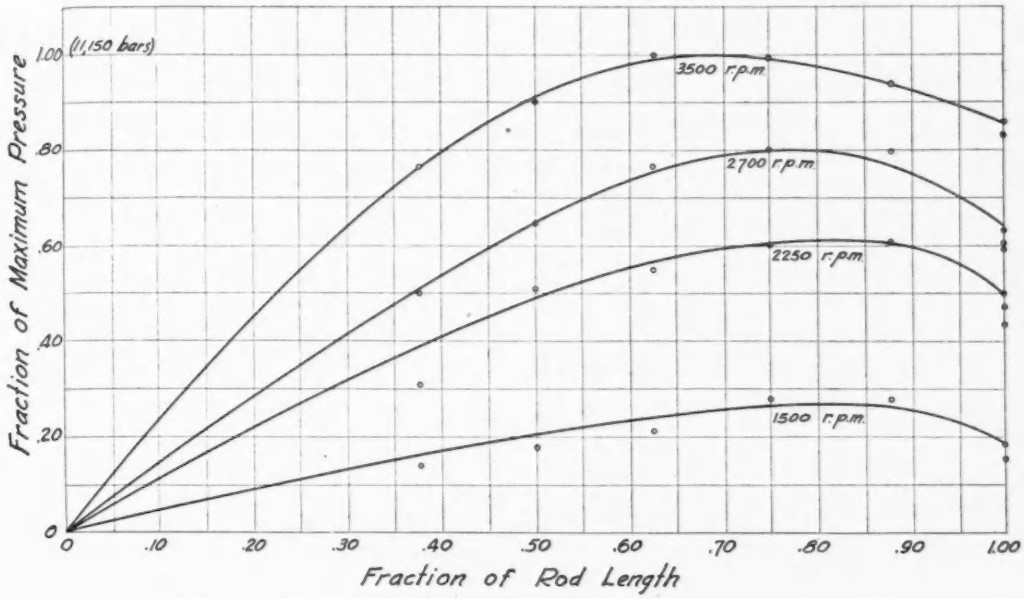

FIG. 6. Distribution of pressure field along 12" rod at four speeds of rotation.

width necessary for this purpose is a measure of the slowness of change of the vortex configurations and may be determined by trial and error.

TABLE IV. *Computed values of V.*

(These quantities apply to one-half the total length of rod.)

1 Fractional length of rod	2 Frequency cycles	3 Acoustical power (micro-watts)	4 Sound energy per vortex (ergs)	5 Volume of air used in sound emission (cm^3/sec.)	6 Volume of one vortex (cm^3)	7 Approximate linear dimensions of one vortex (cm)
0.187	200	0.0145	0.00072	3.93	0.0200	0.271
.234	250	.0270	.00103	4.30	.0171	.258
.280	300	.0670	.00223	5.66	.0188	.264
.327	350	.1485	.00425	7.21	.0206	.273
.374	400	.3575	.00890	9.84	.0246	.289
.420	450	.2385	.00530	7.07	.0157	.251
.467	500	.2485	.00497	5.82	.0131	.235
.514	550	.2950	.00535	6.44	.0117	.227
.561	600	.2635	.00440	5.63	.0094	.209
.607	650	1.4850	.02280	12.20	.0188	.264
.654	700	1.8000	.02575	12.50	.0178	.261
.701	750	2.9750	.03965	15.10	.0200	.271
.746	800	1.9800	.02475	11.50	.0144	.243
.795	850	.7750	.00912	6.79	.0079	.199
.841	900	.8250	.00917	6.63	.0073	.194
.887	950	.3785	.00398	4.22	.0044	.163
.935	1000	.3175	.00317	3.68	.0037	.153
.982	1050	.1025	.00097	2.00	.0018	.122
1.030	1100	.0710	.00065	1.58	.0014	.141
1.075	1150	.1170	.00101	1.58	.0013	.107
1.120	1200	.0585	.00048	1.31	.0011	.102
		12.545				
		×2=				
		25.09				

In this way it may be determined that a band width of about 50 cycles will yield correct values of power per band within experimental error. Table IV shows the distribution of numerous quantities along the rod from hub to tip, all based upon a 50-cycle band width for the power distribution. The values for acoustical power are for the band width of 25 cycles on each side of the frequency in the previous column. The sum of all the powers is 25.09 microwatts, a difference of 4 percent from the total as determined by a microphone. A slightly different band width could be found such that the sum of the powers would amount to exactly 24 microwatts but the accuracy of power measurement does not justify such a procedure.

A division of the power output in any band by the mean frequency over the band will yield a value for the average sound energy associated with each vortex. Such values in ergs are shown in column 4. The energy per vortex shows a maximum at the same place where the power output is a maximum, *viz.*, at 0.7 the length of the rod.

APPARENT SIZE OF VORTICES

A knowledge of the acoustical output of any source enables an estimate to be made of the rate of introduction of fluid, in this case air, at

the source. If a source is emitting a power P at a frequency f, when located near the ground,

$$P=(\pi\rho/2c)f^2V^2 \text{ ergs/second},$$

where V is the volume of fluid used per second; ρ, the density of air; c, the velocity of sound. A solution for V yields

$$V=(2930/f)P^{\frac{1}{2}} \text{ cc/sec.}$$

Values of the quantity V are tabulated in Table IV for half the length of the rod.

It is evident that the volume of air per vortex may be obtained by division of column 5 by the frequency. Column 6 gives these values. Inspection of column 6 reveals the interesting fact that the volume of a vortex does not change a great deal along the rod. Finally, an idea of the order of magnitude of the linear dimensions of a vortex may be obtained by extracting the cube root of the volume of a vortex. These figures are shown in column 7. It is seen that from the hub out to $0.8l$ the size of the vortices is nearly constant and of the order of magnitude of a few millimeters. The size decreases from this point to the tip, as far as can be judged from the emission of sound.

The Effect of Distance in the Broadcasting Studio

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1. INTRODUCTION

IN varying the distance between the source of sound and the microphone, the listener has an impression quite different from that obtained by immediate listening; namely, in moving the source away from the microphone, the sound becomes hollower and somewhat dull, voiceless as to its timbre, whereas when drawn nearer to the microphone the contrary occurs: the sound becomes less hollow, and becomes more distinct and of clearer timbre.

To determine the qualitative changes of sound, observers were asked for the difference of sound which they were able to observe during the alternating recessions of the source of sound (in this case of a singer) from the microphone, and approaches nearer to the latter. They gave the following answers:

On drawing nearer the sound becomes clearer, dryer, shriller, narrower.

On moving away it becomes denser, more compact, wider, hollower ("as if coming from under a dome").

The changes in sound as described above are of great importance both in broadcasting and talking pictures. Far from being a casual factor in the whole complex of sound perception, it is on the contrary an element of great importance which determines the essential character of sound transmitted over the electrical line.

The study of this phenomenon which is called by the author "*the effect of distance*," is, consequently, of great importance in the practical work of broadcasting and is highly interesting from a purely theoretical point of view.

Analyzing the impression obtained from the variation of distance between the source of sound and the microphone, one may conclude that this impression is caused by a change in timbre and in reverberation. It seems to the listener as if the *dimension* of the room where the sound is being transmitted grows larger with the recession of the source of sound.

Applied to talking pictures or play broadcasts a knowledge of the nature and limits of this effect enables one to make an acoustical outline of the stage and to avoid a discrepancy between the acoustic and optic character of the room. In musical broadcasting the effect of distance must be considered when placing the orchestra in the studio.

The effect of distance is caused by a *change in relation* between the *direct waves* (i.e., which emerge immediately from the source of sound), and the *reverberant waves*; a change that is due to a variation of distance between the source of sound and the microphone.

The quantity of energy of direct waves decreases with the distance from the source of sound, while the quantity of energy of reverberant waves may be considered constant, without taking into account irregularities in the sound field.

By considering that the reverberation process consists of the instantaneous decrease of energy of the direct waves, and of the following gradual decrease of energy of the reflected ones, the change of relation between the energies of the direct and reflected waves varies the time of decay of sound energy by 60 db as shown by the microphone, the rate of decay remaining constant. See Fig. 1.

At a shorter distance between the source of sound and the microphone (a) the sound intensity of the direct waves (I_d) is greater than at a longer distance (b) under the same general sound intensity, Fig. 1.

At the same time and for the same reason (change in relation between direct and reverberant waves picked up by the microphone, conditioned by increase or decrease of distance between the source of sound and the microphone), the timbre of sound likewise undergoes a change, because the timbre of reverberant waves is different from that of direct ones, in consequence of irregularities of absorption and directional action of the microphone.

Because of the presence of the *binaural effect*, the listener is able under ordinary conditions to isolate the reflected (reverberant) waves from

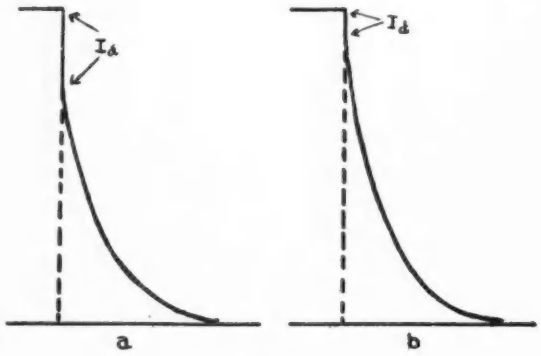

FIG. 1.

the direct ones (we obviously have here some subconscious process). In perceiving a sound transmitted over the electrical line, the binaural effect, as is known, is *absent*, so that the listener is unable to isolate the reverberation from direct waves, the decay forms an immediate continuation of the sound by which it was called forth.

Evidently, this brings out the peculiar "liveness," specific for a transmittance over the electrical line variations of quality of sound so uncommon to hearing under normal conditions and apparent variations of dimensions of the room—all that we call the effect of distance.

2. EXPERIMENTAL

The object of the experiments was to determine the minimum perceptible variation (*threshhold of distinction*) of distance between the source of sound and the microphone valid for various sources of sound.

The work was carried out at a radio studio with a length of about 20 m, having a volume of 1000 cu. m. The reverberation time of the studio for 500 c.p.s. was equal to 0.98 sec. The transmission was carried out through a condenser-microphone, and an amplifier of superior quality. The observers, who listened by means of headphones in a special room where they could not see what was going on in the studio, were given the same musical or oral piece twice, while the distance between the performer and microphone was either changed arbitrarily, or remained constant, in order to check the attention of the observers. The observers were asked whether they had noticed any change in the character of sound in the repeated performance of the piece as compared with the first.

Since the total loudness of sound decreases with the increase of distance between the artist and the microphone, and the resulted changes of distance can be discerned neither by the change of timbral nor the reverberational element, but simply by the difference in the loudness, the said decrease of loudness was compensated for by an *increase of amplification* so that the loudness of sound on the headphones of the observers was kept constant.[1]

The data obtained from these experiments involving violin, singing and speech, are shown in Tables I, II and III.

As can be easily seen in Tables I, II and III, the percentage of recognition grows with the

[1] The compensation by increase of amplification by no means breaks the ordinary conditions of broadcasting since such a compensation is performed by a person controlling the broadcasting, in order to obtain a necessary average loudness at any distance between the performer and the microphone.

TABLE I. *Violin.*

CHANGE OF DISTANCE	% OF DISTINCTION	CHANGE OF DISTANCE	% OF DISTINCTION
from 1 to 2 m	44	from 4 to 6 m	54
1 to 3	73	4 to 7	58
1 to 4	73	4 to 10	77
2 to 3	55	4 to 11	85
2 to 4	57	7 to 10	60
2 to 6	68	7 to 13	70
2 to 7	71	7 to 15	85
2 to 9	87		

TABLE II. *Singing.*

CHANGE OF DISTANCE	% OF DISTINCTION
from 1 to 2 m	66
1 to 3	94
1 to 4	100
2 to 3	50
2 to 4	78
2 to 5	91
4 to 6	69
4 to 7	69
4 to 10	86
7 to 10	63
7 to 13	81

TABLE III. *Speech.*

CHANGE OF DISTANCE	% OF DISTINCTION
from 1 to 2 m	77
1 to 3	88
1 to 4	96
2 to 3	50
2 to 4	83
2 to 5	89
4 to 5	44
4 to 6	59
4 to 7	66
4 to 10	100
7 to 10	45
7 to 11	52

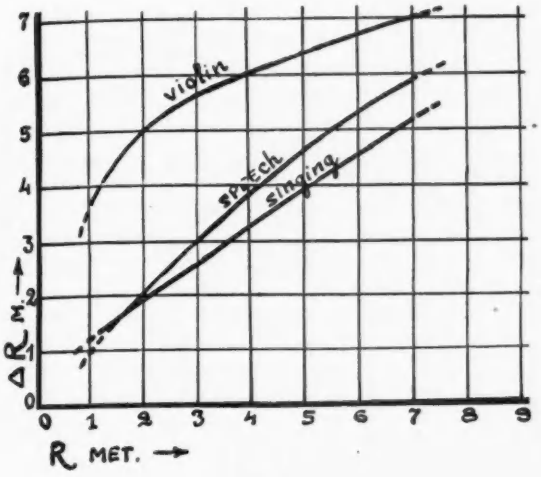

FIG. 2.

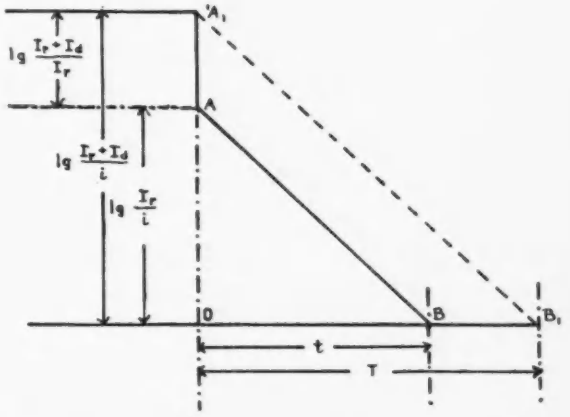

FIG. 3. I_d, sound intensity of direct waves; I_r, sound intensity of reflected waves; i, threshold of audibility; T, time of reverberation (time of decay by 60 db far away from the source of sound).

increase of change in distance between the performer and the microphone and drops with the increase of the initial distance.

Let us consider the recognition realized (i.e., being of some real importance for the radio audience) if it comes up to 75 percent and let us call this value *the threshold of distinction* of the change of distance.

Then, by means of interpolating intermediate values, we can work out a graph, in which the initial distance between the source of sound and the microphone (R) is shown on the axis of abscissas, while the necessary increase of this distance (ΔR) for obtaining the threshold of distinction is indicated on the axis of ordinates (Fig. 2).

3. Discussion of the Obtained Data

The time of decay of sound energy to 60 db, picked up by microphone, can be defined (Fig. 3) as a function of distance between the microphone and the source of sound.

From the similarity of the triangles AOB and A_1OB, it follows:

$$\frac{t}{T}=\frac{\lg (I_r/i)}{\lg [(I_d+I_r)/i]}$$

and considering we assumed that

$$10 \lg [(I_d+I_r)/i]=60;$$

$$t=(T/6) \lg (I_r/i). \qquad (1)$$

For convenience of the final formula, let us express $\lg(I_r/i)$ in the following form:

$$\lg \frac{I_r}{i}=\lg \frac{I_r(I_d+I_r)}{i(I_d+I_r)}=\lg \frac{I_d+I_r}{i}$$

$$-\lg \frac{I_d+I_r}{I_r}=6-\lg \frac{I_d+I_r}{I_r}.$$

Substituting the obtained expression in formula (1) we have

$$t=T\left(1-\frac{\lg [(I_d+I_r)/I_r]}{6}\right). \qquad (2)$$

In case of a spherical wave,

$$I_d=A/4\pi R^2 \qquad (3)$$

where A represents the power of the source of sound, and R the distance between the source and the microphone.

The intensity of sound of reverberant waves may be deduced from the density of the energy of sound E at a steady state if we assume that the deduction made by Buckingham[2] for the radiation of energy on the surface of the wall is applicable also for the surface of the microphone; namely, that the flux of sound energy received per one second by a surface unit, i.e., the intensity of the sound is equal to $\frac{1}{4}Ec$.

[2] Buckingham, Bur. Stand. Sci. Pap. No. 506.

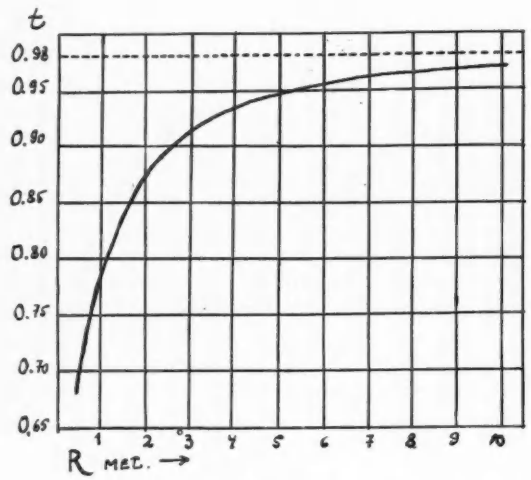

FIG. 4.

By substituting the value E from formula:

$$E=(4A/ac)(1-\alpha),^*$$

where a is the total sound absorption, α, the coefficient of absorption, c, the velocity of sound, we obtain:

$$I_r=(A/a)(1-\alpha). \tag{4}$$

Substituting (3) and (4) in formula (2) we have:

$$t=T\left[1-\frac{\lg\,(1-a/4\pi R^2(1-\alpha))}{6}\right]. \tag{5}$$

The curve of dependence t from R, under the values $T=0.98$, $a=150$ and $\alpha=0.21$ (conditions of the studio, in which the tests have been carried out) is shown in Fig. 4. We see that after 2 meters the curve rises very slowly, from 2 to 7 meters t increases not more than 10 percent.

Considering that such small variations are detected by the ear with difficulty, it is obvious that they do not explain the effect of distance.

It remains to assume that this effect is bound with the immediate perception of the relation of energies of direct and reflected waves, or, which is more correct, of the relation of *varying sound intensity of direct waves* to *the total sound intensity*, which remains constant throughout the test.

* The density of the sound energy of reverberant waves only.

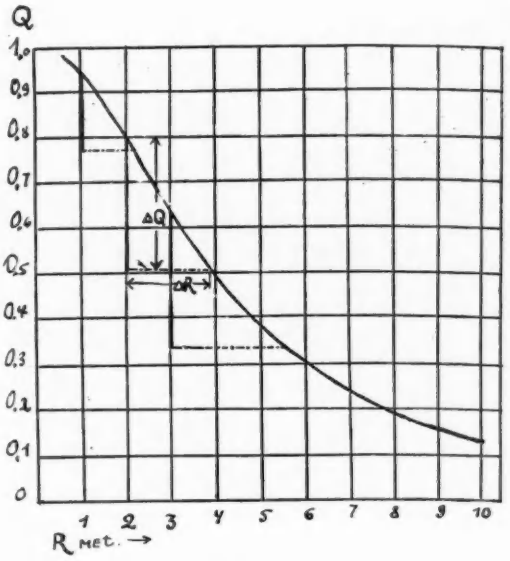

FIG. 5.

This relation—let us express it by Q—is obtained from I_d and I_r, which have been determined before.

$$Q=I_d/I_d+I_r=a/[a+4\pi R^2(1-\alpha)]. \tag{6}$$

The curve of the function Q under values a and α mentioned above (Fig. 5) does not tend to be constant even under greater values of R.

From the experimental curves of singing, violin and speech (Fig. 2) let us take the values ΔR, for $R=1, 2, 3 \cdots$, etc., meters. These values are to be plotted correspondingly from 1, 2, 3 meters on the axis of abscissas in Fig. 5. This figure illustrates the case of singing only, as taken for example. Let us find the increments of the function Q which correspond to those of R, necessary to reach the threshold of distinction. Next let us take the ratios of the increments of the function Q thus obtained to the values of the function itself which correspond to $R=1$, 2, 3, 4, 5, 6, 7 meters (see Table IV). It is obvious that the ratio $\Delta Q/Q$ at distances more than 2–3 m is approximately constant, its value being different for different sources of sound.

Thus in order to discern the change in distance between the source of sound and the microphone, it is necessary for each source of sound to have a certain relative alteration of the value Q.

In conclusion, it should be said that the problem of the effect of distance has not yet been

TABLE IV.

R	1	2	3	4	5	6	7
$\frac{\Delta Q}{Q}$, singing	0.19	0.37	0.48	0.54	0.57	0.59	0.60
$\frac{\Delta Q}{Q}$, speech	0.16	0.38	0.61	0.60	0.63	0.64	0.64
$\frac{\Delta Q}{Q}$, violin	0.54	0.70	0.73	0.73	0.73	0.71	0.70

sufficiently elucidated in acoustic literature. There are but a few observations of the correlation of direct and reverberant waves in the papers of Maxfield, Meyer, Békésy, Olson, Steinberg and Snow.[3] In his extremely interesting paper on the perception of increase of sound and the reverberation, Békésy attributes the effect produced by a change of correlation between the energy of direct and reverberant waves to the moment of growth of sound.

This work was carried out by the group for scientific research, and I take this opportunity to express my sincere acknowledgment to the leader of the group, Professor S. N. Rshevkin whose valuable advice and instruction were of great help during the process of this work.

[3] T. Maxfield, Trans. Soc. Mot. Pict. Eng. **14** (1930). E. Meyer, E.N.T. **4**, 135 (1927). G. v. Békésy, Ann. d. Physik **8**, 7 (1931). G. v. Békésy, Ann. d. Physik **16**, 7 (1933). H. Olson, Proc. I.R.E. **21**, 3655 (1933). I. Steinberg and W. Snow, Elec. Eng. **53**, 12 (1934).

The Noise Problem in the Application of Fans

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INTRODUCTION

THE increasing use of air-conditioning apparatus in homes and offices has introduced new acoustical problems. The magnitude of such problems depends upon the operating functions of the apparatus considered with the noise level in which it must operate, and the personal element involved in the objectionableness of a particular value and character of noise. Recently, the latter has received much attention, and comes under the general subject of physiological effects[1] of noise. The functions of an air-conditioning system are heating, cooling, humidifying, dehumidifying, cleaning and circulating of air. The circulation function in some form is a necessary element in all the other functions. To provide this circulation, fans of some type are almost universally used.

The importance of the noise problem in the application of fans is indicated by a statement of two facts: First, the air circulation is the most important single function in any air-conditioning system; and second, the fans which provide the circulation are always in a more intimate contact with the conditioned space than any other element.

This paper deals with the noise problems in the application of fans to provide air circulation in air-conditioning apparatus and kindred devices. It considers the sources of such noise as may be traced directly or indirectly to the fan and its immediate system, or that noise for which the fan may be held responsible. Particular consideration is given to the propeller type of fan for this service, but practically all the information applies equally well to other types of fans.

Several illustrations of different types of units are given as practical examples of various classes of noise commonly found in the application of fans to air-conditioning apparatus.

[1] Donald A. Laird, *Noise*, Sci. Am. **139**, 508–510 (1928); Juichi Obata, Sakae Morita, Kin-ichi Hirose and Hiroshi Matsumoto, *The Effects of Noise upon Human Efficiency*, J. Acous. Soc. Am. **5**, 255–261 (1934).

SOURCES OF FAN NOISE

The sources of what is commonly known as fan noise may be classified as inherent fan noise and other sources which modify it. The inherent noise conveniently represents a base noise level from which to evaluate the noise due to other sources. This may be defined as the noise of a particular fan under the best possible operating conditions. Usually it is not possible to obtain these ideal conditions in a piece of apparatus. Thus, a consideration of other sources which modify the inherent noise is of equal importance to that of the inherent noise.

The five classes of fan noise which modify the inherent noise of a fan may be listed as follows:

(1) Noise increase due to improper flow conditions.
(2) Noise increase due to improper application or aerodynamic design of fan.
(3) Change of inherent noise by other parts of the apparatus.
(4) Amplification by the fan of noise made by other parts of the apparatus.
(5) Noise increase due to improper mechanical design or construction.

NOISE STANDARD

The decibel unit used throughout this paper is a frequency weighted logarithmic unit with 0.001 bar sound pressure at 1000 cycles for zero reference.

This does not agree with the latest standards proposed by the American Standards Association. However, several of the fans whose ratings have been specified according to the 0.001 bar at 1000 cycles zero reference have been measured with a noise meter calibrated to read according to the latest proposed noise meter standards.[2] The noise readings are increased approximately 10 decibels with this new calibration.

INHERENT FAN NOISE

Recently, notable progress has been made toward the development of fans having a low

[2] R. G. McCurdy, *Standardization of Noise Meters*, Elec. Eng. **54**, Jan. 14–15 (1935).

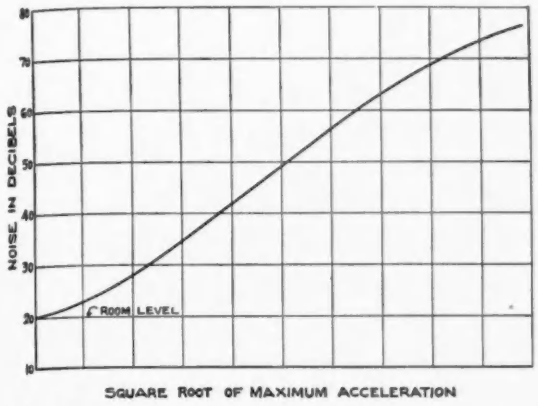

FIG. 1. Characteristic curve of the acceleration component of noise for any propeller fan.

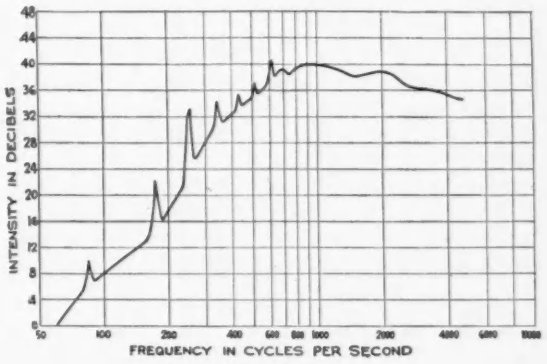

FIG. 2. Noise intensity—frequency curve for an Aphonic fan.

inherent noise[3, 4] and the specification of ratings for such fans by their manufacturers. However, there remains much to be done on this. The reluctance definitely to specify noise ratings may be explained by the continued nomadic tendencies of noise standards and difficulty in properly allocating the noise of the complete apparatus to the proper sources. The sources of noise which add to the inherent fan noise are numerous and without their proper understanding noise ratings may be subject to controversy.

It has been found that the inherent noise of any fan depends largely upon two distinct components, an air acceleration component and a blade frequency component. The magnitude of each of these components depends upon the design of fan and the service for which it is intended.

For any propeller type fan, the acceleration component has been determined empirically[4] as a definite function of the square root of the maximum acceleration of the air as it passes across the fan blades. This function is entirely independent of such factors as number of blades, speed, diameter, blade shape, etc., except as these factors affect the air acceleration. Fig. 1 shows the characteristic shape of this component noise curve for any propeller fan. For the average fan the upper end of the curve represents a peripheral velocity of about 13,000 feet per minute. This is considerably above normal application practice so that the useful range occurs over the straight portion of the curve where noise is directly proportional to the square root of the maximum air acceleration.

The blade frequency component is composed of blade frequency and its harmonics, with the higher harmonics usually predominating. This component is responsible for the familiar "whine" so objectionable in some high speed propellers, and the "flutter" in low speed fans.

In a well-designed propeller fan at ordinary speeds, the blade frequency component is small as compared to the acceleration component and is practically negligible in evaluating the inherent noise. Fig. 2 shows the frequency-intensity curve for a 3-blade 16-inch diameter, G.E. Aphonic fan operating at 1725 revolutions per minute against a static pressure corresponding to that at maximum efficiency. The fundamental blade frequency is 86.3 cycles per second, and peak values may be noted for all harmonics up to and including the seventh with the third predominating. The total noise reading is 56 decibels which is considerably above the highest peak value.

The total noise of a particular design of fan may be expressed empirically as a logarithmic function of the peripheral velocity. Fig. 3 shows such a test curve for a 17-inch diameter, 3-blade, low pitch Aphonic fan operating at maximum efficiency. The curve is expressed by the equation,

$$db = 67 \log U - 199.5.$$

For a particular series of similar fans, the noise for a given peripheral velocity often varies

[3] L. S. Marks and J. R. Weske, *Design and Performance of Axial Flow Fans*, Trans. A.S.M.E. **56**, 807–813 (1934).

[4] K. D. McMahan, *Development of Quiet Propeller Fans*, Gen. Elec. Rev. **37**, 82-86 (1934).

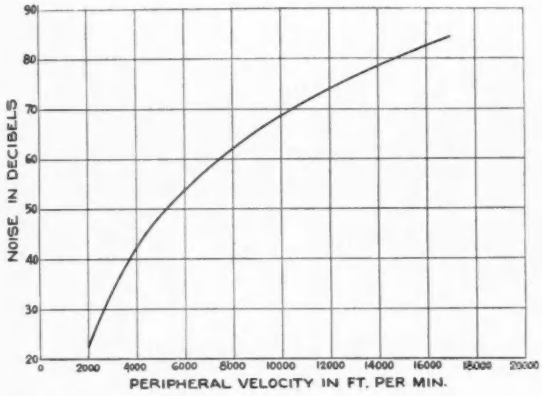

FIG. 3. Noise—peripheral velocity curve of a 17-in. diameter Aphonic fan.

slightly with fan diameter. Considering this factor, the expression of noise, for $d^{1.25}N > 15{,}000$, for a whole line of geometrically similar fans of the design given above becomes,

$$db = 67 \log_{10} (0.129 d^{1.25} N) - 199.5$$

where d = fan diameter, in inches
db = noise in decibels
N = speed in revolutions per minute
U = peripheral velocity in feet per minute.

This expression together with the other performance characteristics may be represented in chart form as indicated by Fig. 4. Here static pressure is plotted as abscissa and fan diameter as ordinate on a logarithmic scale. Air flow, speed, horsepower and noise are all represented by intersecting straight lines on this scale. The intersection of any two lines representing two performance characteristics determines the remaining ones. It may be noted that noise depends principally upon the static pressure against which the fan must operate. Such a chart is very useful in the application of fans, and shows at a glance the true inherent noise of the particular fan represented.

NOISE DUE TO IMPROPER FLOW CONDITIONS

A fan intended for low inherent noise must be designed for a particular pattern of air flow into the fan and across its blades. For this fan, the discharge flow pattern will also be fixed. The propeller type has a rotational component of velocity in the discharge air stream, the magnitude of which depends largely upon the pitch of the fan. A disturbance of these patterns changes the flow distribution across the fan blades and thus increases the inherent noise. Either of the two components of noise may be affected by such a disturbance. Any fan is much more critical to obstructions of its inlet than discharge.

In general, a concentric, circumferential obstruction of the fan inlet increases the acceleration component of noise. An unsymmetrical ob-

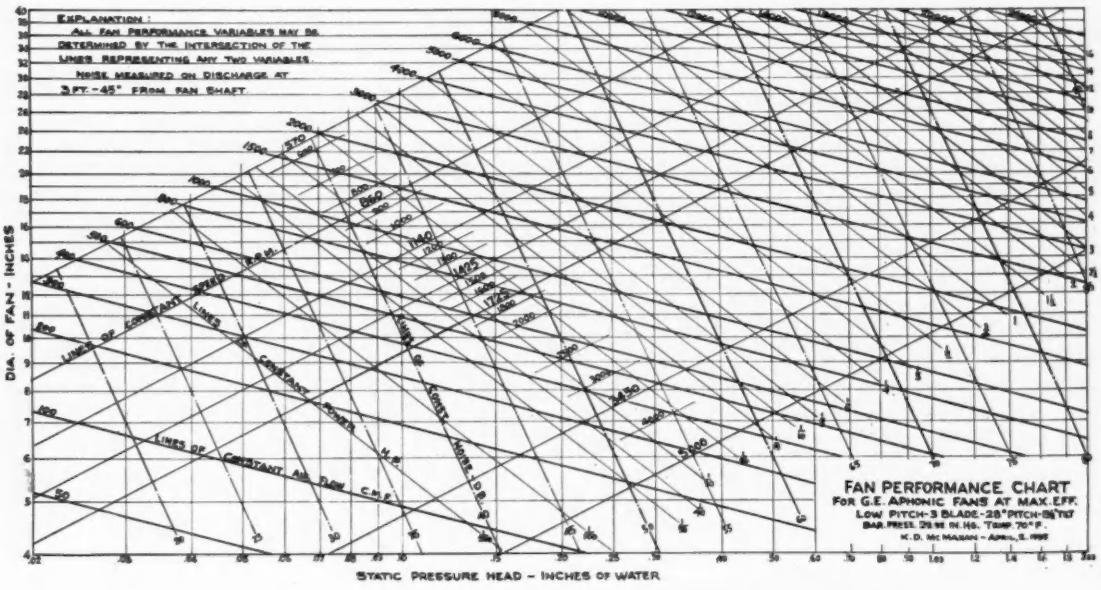
EXPLANATION:
ALL FAN PERFORMANCE VARIABLES MAY BE DETERMINED BY THE INTERSECTION OF THE LINES REPRESENTING ANY TWO VARIABLES.
NOISE MEASURED ON DISCHARGE AT 3 FT. - 45° FROM FAN SHAFT
DIA. OF FAN - INCHES
STATIC PRESSURE HEAD - INCHES OF WATER
FAN PERFORMANCE CHART
FOR G.E. APHONIC FANS AT MAX. EFF.

FIG. 4. Performance chart for G.E. low pitch, Aphonic fans at maximum efficiency.

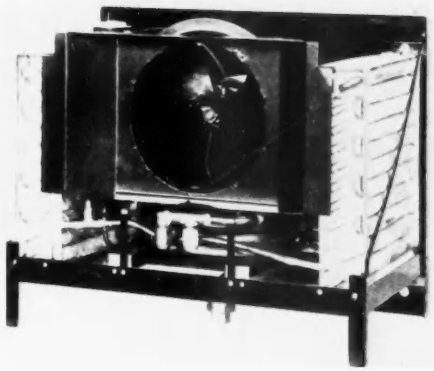

FIG. 5. G.E. wall-mounted room cooler with cabinet removed to show the fan assembly.

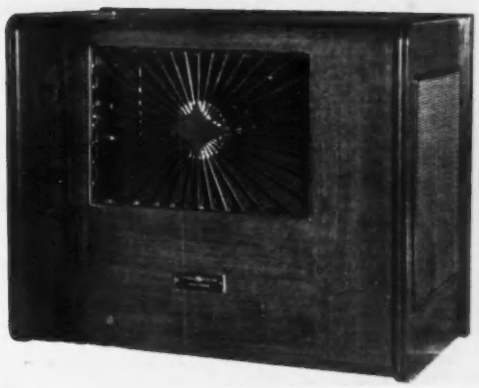

FIG. 6. G.E. wall-mounted, room cooler with cabinet to show the fan discharge grill.

struction increases the blade frequency component. Even a slight obstruction at the periphery of a fan on the inlet side may increase the intensity of certain blade frequency harmonics as much as 10 or 15 decibels. Motor supports are the most common offenders and should not be allowed to come near the periphery of a fan unless specially shaped to prevent interference with the flow pattern.

An unsymmetrical obstruction on the discharge does not increase the inherent noise in the same proportion as on the inlet but the effect is similar. A symmetrical obstruction at the discharge does not appreciably affect the inherent noise but produces the same apparent result. The fan in such cases performs as though it were working against a static pressure much higher than normal, and has the "ear marks" of an improper fan application. The trouble arises from the rotational component of velocity and may be remedied by the use of straightening vanes.

Fig. 5 shows a G.E. wall-mounted room cooler unit without the cabinet. This unit uses a 10-inch diameter Aphonic fan for air circulation. Air enters the unit from both ends and is discharged directly from the fan at the center. The fan is located so that air enters from all sides without obstruction. The motor mounting is suspended from the back to prevent obstructions. Fig. 6 shows the same unit enclosed in its cabinet. The discharge grill is an important element in this application, and serves a double purpose. It straightens out the rotational component of velocity thus giving the unit a higher projecting velocity, and at the same time offering no obstruction to the air flow from the fan discharge. The total noise of this complete unit in production does not exceed the inherent noise of the fan by more than one decibel.

NOISE DUE TO IMPROPER APPLICATION

A fan, like other devices, is designed to operate over a limited range of conditions. When this range is exceeded some performance characteristics are sacrificed, and this particularly applies to noise.

Fig. 7 shows the performance curves of a 17-inch diameter Aphonic fan operating at 1725 revolutions per minute and performs according to the chart, Fig. 4. The allowable operating range is indicated near maximum efficiency.

To illustrate the effect of improper application, consider Table I. The difference in noise for the same static pressure, and air flow, at different operating points is worthy of note.

TABLE I.

Static Pressure, (in. of water)	Noise, (db)	Air Flow, (C.F.M.)	Fan Diameter, (in.)	Speed, (r.p.m.)	Power, (hp)	Operating Point
0.20	58	2510	17	1725	0.275	*A*—Fig. 7
0.20	55	2510	20	1140	0.197	Max. Eff., Fig. 4
0.40	65	1825	17	1725	0.265	B—Fig. 7
0.40	62.5	1825	14.5	2200	0.277	Max. Eff., Fig. 4

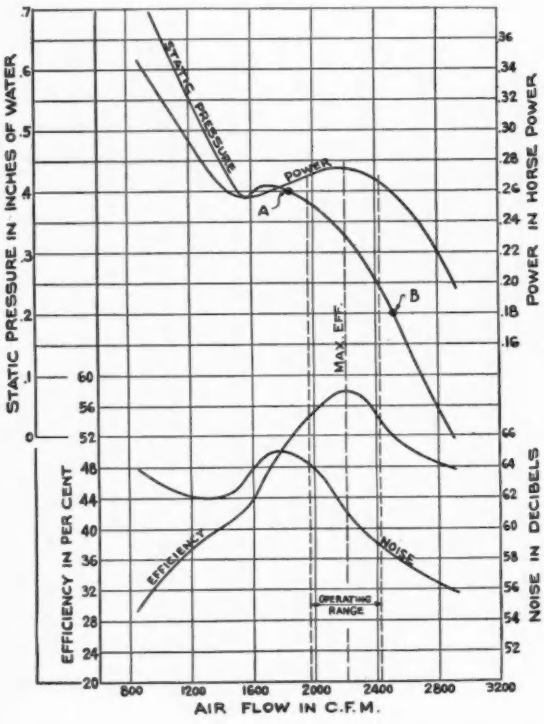

FIG. 7. Performance curves of a 17-in. diameter, low pitch, Aphonic fan at 1725 r.p.m.

Figs. 8 and 9 show an oil-cooler unit made by the General Electric X-ray Corporation before and after changes in the application of the fan. Fig. 8 shows the fan arranged to push air through the radiator section with no cowling surrounding the fan. Under operating conditions the noise of this unit was 65 decibels with a power input to the fan of 1/4 horsepower for a given air circulation. Fig. 9 shows a similar unit with a smaller diameter and speed of fan, with cowling, arranged to draw the same volume of air through the radiator. With this application the noise is reduced 18 decibels to 47 decibels and the power from 1/4 to 1/20 horsepower. The inherent noise of the two fans involved is not far different so that practically the entire reduction in noise is due to proper fitting of the fan to the job.

NOISE CHANGE DUE TO OTHER PARTS OF APPARATUS

The inherent noise of a fan or some component of it may be amplified by other parts of the apparatus sometimes remotely located from the

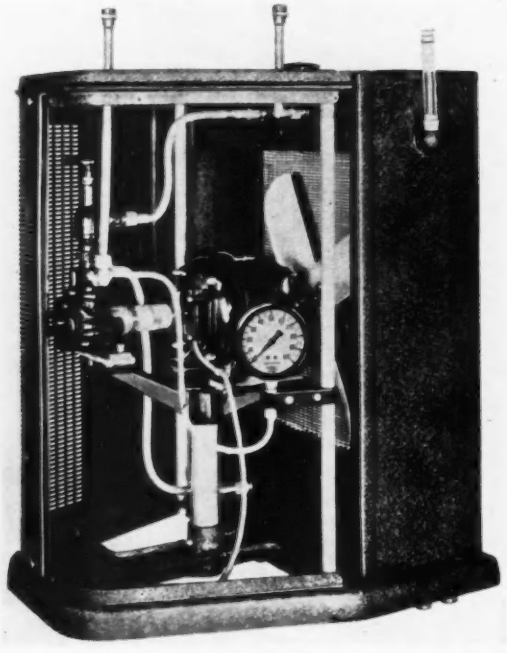

FIG. 8. G.E. X-ray Corporation oil cooler before changing the fan.

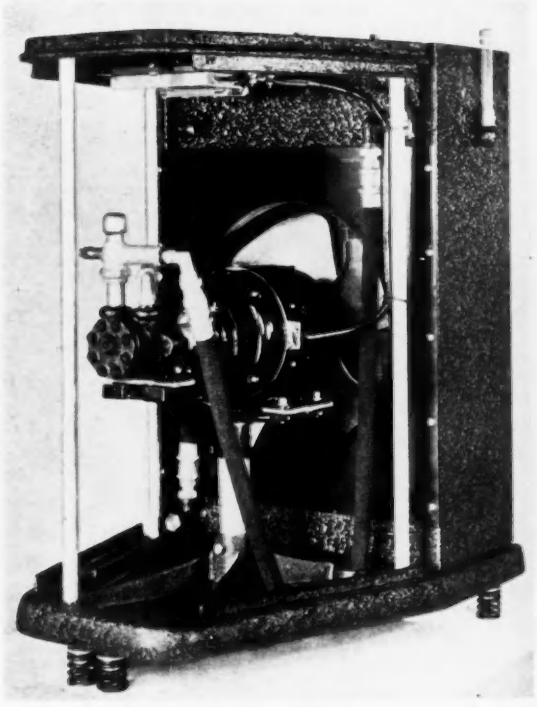

FIG. 9. G.E. X-ray Corporation oil cooler after application of an Aphonic fan.

fan. This type of noise is closely related to and often confused with what is known as "duct noise." This noise increase results from the amplification of certain components of fan noise by parts of the duct system whereas "duct noise" is that generated within the system, usually a function of air velocity and has no relation to the inherent noise of the fan. This also applies to some types of grills, deflecting vanes, dampers, and similar devices. In the former case, a change of fan having the same inherent noise but made up of different characteristic frequencies would change the resultant noise, while in the latter case, such a fan change would in no way affect the resultant noise.

It is beyond the scope of this paper to discuss "duct noise" but it is timely to suggest that the wide disagreement of published results on this subject may find significant explanation in fan noise or amplifications of it.

The increase in inherent fan noise by other parts of the apparatus may represent an increase of either the blade frequency or acceleration component. The type of amplification usually depends upon the method by which the parts respond to the stimulus set up by the fan. The system may respond as a mechanical vibration of the parts or as an acoustical vibration of the air columns. The latter usually occurs at blade frequency or harmonics of blade frequency. Mechanical vibration may happen at any frequency. In this connection, it should be noted that a fan contains all frequencies over a wide range with small changes in intensity and therefore supplies "shock excitation" to the system. Thus, mechanical resonance of any parts of the system is not limited to any particular exciting frequency.

The character of the noise obtained from these sources corresponds exactly with that of the fan. This fact makes it very difficult to distinguish noise from the above sources, without a frequency analysis.

In a certain piece of apparatus, it was found by frequency analysis that some of its sections were responding acoustically to the fan blade frequency and upper harmonics. A comparison of the intensity frequency curves for the inherent fan noise, and the same fan in this particular unit is shown by Fig. 10. The 3-blade fan operated at 1725 revolutions per minute and emitted a fundamental blade frequency of 86.3 cycles per second. The fundamental was amplified 17 decibels, the second harmonic 9 decibels, the third only slightly, and the remainder of the curve none at all. Further tests showed that this unit would respond to any fundamental or harmonic between 80 and 240 cycles per second. This made it necessary to change the design of the unit before any fan of the desired design could be properly applied.

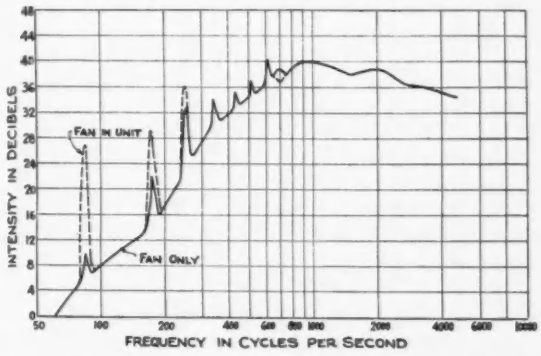

FIG. 10. Comparative noise intensity curves to show the amplification by the unit of the fan blade frequency and harmonics.

The general subject of sound absorption also falls in this classification of noise. By the use of sound absorbing material, the resultant noise may be reduced below the inherent noise of the fan, and represents the only case where this arbitrary base level may be exceeded in applications. However, from the viewpoint of the author, the use of this material should be resorted to after all elements in the fan application have been considered, and hence does not represent a problem in fan application.

AMPLIFICATION BY THE FAN OF NOISE FROM OTHER PARTS OF APPARATUS

A fan itself may not be the source of a particular noise, but it may amplify that noise produced by some other part of the apparatus. When such a noise source may be traced to the characteristics of the fan, it becomes just as much a problem in fan application as if the noise originated in the fan. The origin of this class of noise is usually in the driving motor or other power transmission mechanism. The amplification may be due either to resonant or to forced vibration of

the fan parts, and corresponds to some inherent noise component of the driving mechanism. Usually, however, resonance of some type is involved.

The most common source of this class of noise is what may be termed "blade resonance," and as the name implies, is a resonant vibration of the fan blades excited by the electrical characteristics or torque pulsations of the driving motor. It may be recognized by its frequency which is double the impressed a.c. frequency of the motor, 120 cycles for 60-cycle motors, or its harmonics. The problem may be solved by one of three methods: (1) The isolation of the motor shaft from the fan by a suitable elastic hub; (2) the changing of the resonant frequency of the fan so that it does not agree with that of the motor; or (3) the introduction of sufficient damping in the fan blades to prevent the vibration from reaching objectionable proportions. The first method eliminates any possibility of trouble by preventing the exciting force from reaching the blades, but it often presents mechanical difficulties especially with large, high speed fans. The second method is just as satisfactory, but involves cut-and-try methods of changing the thickness or material of the blades so as to shift the resonant frequency. The third method is less satisfactory in that the exciting force and the resonant condition are still present. The damping may be provided by coating the blade with some material or by laminating the blade and spot-welding the laminations together. The importance of this source of noise cannot be overestimated as it forms the basis of a majority of fan motor noise complaints. The noise is of the same frequency and sounds to the casual observer as motor noise, but the motor is in no way responsible.

A noise source known as motor "end bump" also comes in this classification of noise. The term implies the exact cause of the noise. The motor shaft continually bumps the thrust bearing and emits a noise which varies over a wide range of frequencies from a few to several hundred per second depending upon the length of "end play." The fan is not responsible for this type of noise, except as the condition may be aggravated by the weight or unbalance of the fan, but it is easily confused with blade resonance.

Noise Due to Mechanical Construction or Design

In this classification come the sources of noise which are due to faulty or improper construction and designs which may not be fitted for the service intended.

Loose parts, poor hub fits, and improper alignment of blades, increase the inherent noise. They are of minor importance as many defects are eliminated in inspection and test of the apparatus and never reach actual service in such condition.

Loose parts almost invariably change their resonant frequencies so that they correspond to that of the driving motor. The noise thus obtained is similar to "blade resonance." Improper blade alignment increases the blade frequency component of noise in much the same manner as improper flow conditions of the unsymmetrical type.

A fan of improper mechanical design may produce many types of vibrations which result in increased noise. A rather uncommon but serious one results in "traveling waves" in the fan. The excitation for such a mode of vibration usually comes from the motor torque pulsations. The noise is increased slightly but the seriousness comes from the fatigue failure of the fan. The only remedy is a mechanical redesign.

Static Pressure and Inherent Fan Noise

The foregoing analysis of the sources of fan noise precluded for purposes of discussion a given design of fan with a fixed inherent noise. Obviously, the selection of a fan with low inherent noise is of equal importance in the problem of fan application. This is primarily a fan research problem and outside the subject of this paper, but the problem of applying a fan of low inherent noise to particular operating conditions is distinctly one of application. An understanding of the factors involved in inherent fan noise materially aids in keeping it to a minimum. The inherent noise of practically any fan operating in a fixed system may be expressed as a logarithmic function of the peripheral velocity of the form,

$$db = C_4 \log_{10} U - C_5$$

and the static pressure of the same fan as

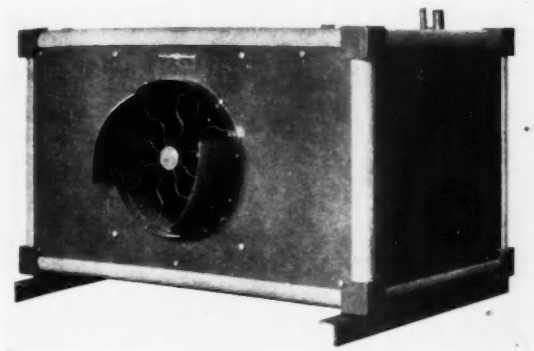

FIG. 11. G.E. store-type room cooler equipped with air deflectors.

$$P_s = C_6 U^2,$$

where P_s = static pressure of fan in inches of water, C_4, C_5, C_6 = constants.

By combining the above equations, the expression of noise as an independent function of static pressure may be expressed as

$$db = C_4 \log_{10} (P_s/C_6)^{\frac{1}{2}} - C_5.$$

For a complete series of fans of similar design, with different diameters and blade angles, the above relation may be slightly modified to include these factors along with the static pressure. However, the static pressure always enters as a higher power than any of the other factors. The conclusion is that inherent noise of any fan depends principally upon the static pressure against which it operates. Indeed, for some types and designs noise depends entirely upon static pressure.

For the Aphonic fan represented by the chart, Fig. 4, the expression for noise is, for $(d^{\frac{1}{4}}P_s^{\frac{1}{2}}) > 0.20$,

$$db = 67 \log_{10} (6.58 \times 10^3 d^{\frac{1}{4}} P_s^{\frac{1}{2}}) - 199.5.$$

The fan diameter enters this expression as the $\frac{1}{4}$ power while the static pressure is raised to the $\frac{1}{2}$ power. Thus, a tenfold increase in diameter, representing a one-hundred-fold change in air flow, increases the noise 15 decibels while a 2.7-fold increase in static pressure is required for an equal increase in noise.

Since the static pressure is of paramount importance in fixing the inherent fan noise, it is essential that precautions be taken to hold it to a minimum in devices required to operate with minimum noise.

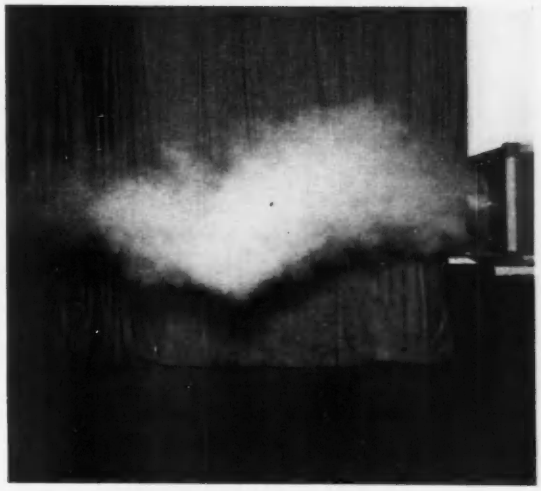

FIG. 12. G.E. store-type room cooler with air deflectors undergoing smoke tests to show air deflected in a horizontal plane.

FIG. 13. Same as (12) except for vertical plane.

Fig. 11 shows a General Electric store-type room cooler equipped with air deflectors on the discharge of the fan. These deflectors give the unit a particular type of discharge which spreads at a wide angle in one plane, and remains narrow in another perpendicular plane. This is very essential in certain installations to prevent drafts and provide cooling over a wide area. Figs. 12

and 13 show smoke tests on this unit with the deflectors adjusted to give a wide diffusion in the horizontal and vertical planes, respectively. This device makes use of the rotational component of discharge velocity to supply the energy for producing the desired results. The static pressure and other performance characteristics including noise are in no way affected by the application of the deflectors. The normal practice of accomplishing this result with a diffusing grill would have required for this particular unit an increase of static pressure sufficient to raise the inherent fan noise approximately 8 decibels.

Noise and Efficiency

Noise and efficiency of any fan application are not related upon any energy basis because of the exceedingly small amount of energy required to produce noise, but they are related upon other considerations. The same factors which produce noise in a fan application tend to reduce the fan efficiency. The increase in noise precedes the loss in efficiency. Noise is therefore a much more critical indicator of proper application than of efficiency.

This may best be illustrated by reference to Fig. 14. Here curves of efficiency and noise are plotted against degree of inlet obstruction near the periphery of a fan. From these curves, it may be noted that the noise increases rapidly with small degrees of obstruction while the efficiency is moderately reduced. With greater obstruction, the noise curve flattens and the efficiency curve begins to drop rapidly. In effect, the noise is a harbinger of impending loss of efficiency.

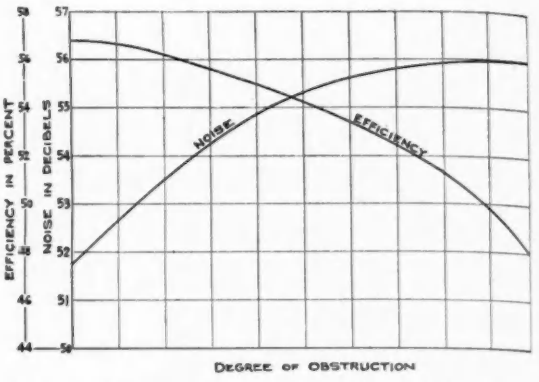

Fig. 14. Comparative curves of noise and efficiency to show the effect of various degrees of obstruction at the fan inlet.

Summary

The noise of any fan may be rigidly specified for particular operating conditions. Likewise, the resultant fan noise of a piece of apparatus employing such a fan, when properly applied, may be accurately predicted. When the noise of the complete apparatus exceeds the noise of the fan alone, the causes of such increase may be definitely allocated to the offending sources. This simplifies the problem of their elimination.

The noise of a fan depends principally upon the static pressure against which it is required to operate. This fact necessitates the holding of the pressure drop in the air flow system to a minimum, and often affects the design of the entire apparatus.

Noise and fan efficiency are not connected by any energy relations, but the same factors which increase noise tend to reduce efficiency. A noise meter is capable of locating inefficiencies in a fan system which a wattmeter cannot detect.

Apparent Duration of Sound Perception and Musical Optimum Reverberation

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(Received September 1, 1935)

I. APPARENT DURATION OF SOUND PERCEPTION

IN one of our works[1] we established a new fact, namely, that the apparent duration of sound impulse depends on its equivalent loudness and is determined by its integral of sound duration:

$$\int_{t_1}^{t_2} \log I' dt,$$

where I' is the sound intensity expressed in terms of the threshold of audibility and t is the actual time of the impulse.[2]

In the indicated work this law was verified only in an individual case of one group of impulses consisting of clicks. In the present article we give the results of experiments with impulses consisting of pure tones of varied duration.

For the experiments a specially constructed vacuum tube, so-called "vacuum-tube impulsator," was used. By means of this apparatus the length of the impulse is determined by the time elapsing between the closing and the opening of the electric current by means of a relay. This relay is actuated by the charge of the condensator, that is, the circuit closes when the condensator is charged up to a definite potential and the circuit opens when the condensator, charging through the given resistance, is discharged to a given potential. Changing the resistance, we can, within great limits, change the time elapsing between the closing and the opening of the circuit, that is, the length of the impulse.

The vacuum tube impulsator (see Fig. 1) makes it possible to receive in telephone T, impulses of a sound frequency N having a duration of 0.012 second and higher, up to any desired point. The sound impulse is delivered to the telephone T by closing the key D inasmuch as the negative potential of the battery B through the key D, coming into the lamp grating, lowers the anode current to a potential P min. (see Fig. 2) and the relay *Rel* shunts the telephone off.

The duration of the contact, effected by means of the key D, is exactly 0.004 sec., and is absolutely constant.

The condensator C is closed at a resistance R_1 through which it discharges after the opening of the key D. As a result of the discharging of the condensator C, the potential of the anode current increases and, when it reaches the critical potential (Pcr), starts the relay *Rel* working, which again shunts the telephone in. In view of the fact that the increase in the anode current follows, sufficiently closely, the law:

$$P = P_{\max} - P_{\min} e^{-T/CR_1},$$

we find that by changing the resistance R_1 we make it possible to regulate the duration of the sound impulse in the telephone T within any limits from 0.012 sec. and higher.

The milliammeter A_1 serves to measure the

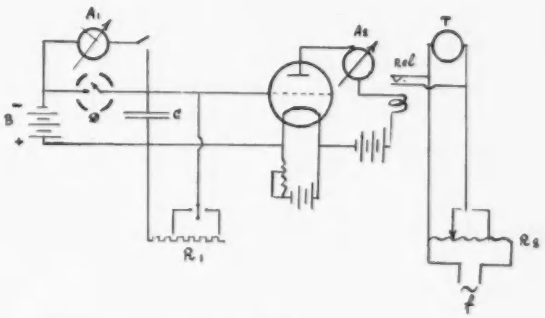

FIG. 1.

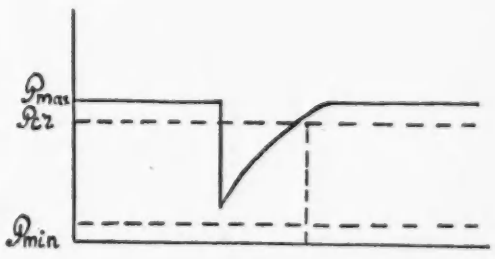

FIG. 2.

[1] Samuel Lifshitz, Zeits. f. Physik **83**, 123 (1933). J. Acous. Soc. Am. **5**, 31 (1933).

[2] As later experiments showed, this law of apparent duration should obviously be present in all our perceptions. At least, insofar as light impulses are concerned, we discovered the same manifestations by experimentation.

resistance R_1. The durations of the sound impulses are determined experimentally and graduated in milliamperes which are measured by the milliammeter.

The milliammeter A_2 serves as a control for the anode current.

The rheostat R_2 serves for the variation in equivalent loudness of the sound impulses to the telephone T.

To determine the integral effect with the help of the impulsator, the one experimented upon was given, over the telephone, two impulses of similar frequency but of varied duration and varied equivalent loudness, and he was supposed to tell whether the apparent duration of both impulses was the same. In case of a negative reply, by means of changing the equivalent loudness of one of the impulses, a point was reached where the one experimented upon stated that the apparent duration of the impulses was the same. According to the milliammeter A, which was graduated as indicated above, the objective durations of each of the impulses per second were determined, and the equivalent loudness in decibels (db) was determined by means of the audiometer.

The first series of experiments was conducted with impulses of tone 1000 vib./sec. An impulse of tone 1000 vib./sec. of a definite equivalent loudness and duration was transmitted into the ear over the telephone. An impulse of the same period but of a different duration was compared with the first, but the equivalent loudness of the second impulse was so selected that the apparent duration of the two impulses seemed alike.

The equivalent loudness in these experiments ranged from 34 to 84 db.[3] Louder impulses resulted in fatiguing the ear. The first impulse was taken from the equivalent loudness of 84 db, and the impulse of greater duration compared with that.

Inasmuch as, in the case in hand, the impulse was part of a tone of constant equivalent loudness, the integral of duration in decibels became a simple product $L_0 t$ where $L_0 = 10 \log I_0'$.

[3] During the experiments for the threshold of audibility 2.510^{-16} watt/cm² was used. The figures for this threshold of audibility were published in the abstract submitted for the 13th meeting of the A.S.A. In the present paper all data are calculated with a threshold of 1.10^{-16} watt/cm².

TABLE I.

K (units of duration)	Initial Impulse db	Initial Impulse Time (sec.)	Limiting Impulse db	Limiting Impulse Time (sec.)
2	84	0.025	34	0.060
5	84	0.060	34	0.150
10	84	0.120	34	0.310
20	84	0.240	34	0.620
32	84	0.380	54	0.600
43	84	0.510	64	0.670
48	84	0.570	70	0.680
53	84	0.630	76	0.690

Within the limits in which the integral law of apparent duration holds true, the apparent duration of each group of the compared impulses is determined by the equation, $L_0 t = K$, where K is the number of units of apparent duration referred to the given group of impulses.

Let us give the following determination of the units: A unit of apparent duration of tone is the apparent duration of tone of 1000 vib./sec., expressed by the product of 1 db×1 sec.

The results of the experiments are given in Table I and Fig. 3, beginning with the very lowest values of apparent duration and ending with the very highest for which the integral law still holds good.

In the first column we have the values of apparent duration K. In the second column the values for the equivalent loudness and actual time of duration for the initial impulse and in the third column, the same for the limiting impulse. By "limiting" here we mean such an impulse of greatest duration which, having the given apparent duration, can be compared with the initial impulse and still have the integral law apply. Thus the table gives the limits in which the law is applicable.

As seen from the table, the integral law holds true within the limits from 2 to 53 units of apparent duration. In addition, up to 20 units of apparent duration the law is applicable for the entire range from 34 to 84 db, i.e., when the actual duration of two impulses differ, one from the other, two and a half times, the impulses become of like apparent duration under proper selection of equivalent loudness.

For cases greater than 20 units of apparent duration the limits of applicability of the integral law become less. For instance, with 48 units of apparent duration two impulses seem alike when

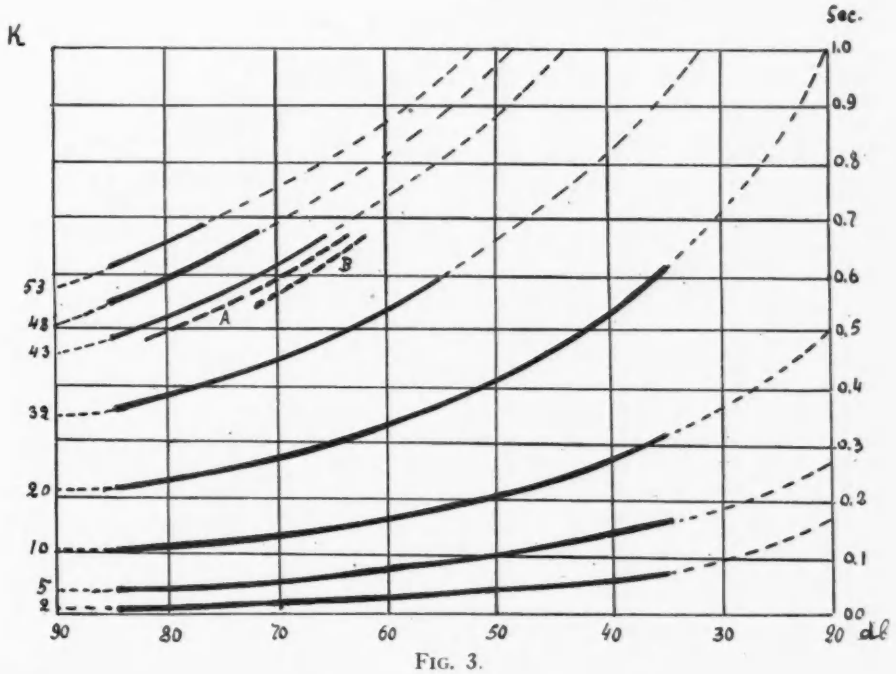

FIG. 3.

their actual durations differ in the ratio of 1 to 1.2. If the impulses vary even more, in order to make them of the same apparent duration it is necessary to alter their equivalent loudness considerably more than is called for in the simple law $L_0 t = K$. Seemingly, the absolute duration of the impulse is of significance here, and the borderline of the application of the integral law is determined by the duration, which should be about 0.7 sec.

In Fig. 3 we have the graphic results of the experiments. The decibels are along the axis of the abscissas; along the axis of the ordinates at the right we have the time in seconds; at the left, the units of apparent duration (K).

The dotted curves represent the theoretical hyperboli according to the equation, $L_0 t = K$.

Each hyperbola corresponds to its value for the constant K, i.e., to the number of units of apparent duration. The heavy curves represent the results of the experiments within the limits of which the integral law of apparent duration is applicable.

All of the foregoing data were secured for tones of 1000 vib./sec. It is interesting to solve the problem as to how far the integral law is applicable in the field of other frequencies and also for instances when two comparable sound impulses have different frequencies. For the purpose of finding a solution to this problem, the following series of experiments was conducted: With the help of a vacuum-tube impulsator two impulses having different frequencies were compared, and the equivalent loudness at which they seemed to have similar duration was determined. One impulse always had a frequency of 1000 vib./sec. and the other impulse a frequency of 50, 100, 200, 500, 2000 and 4000 vib./sec.

The results are given in Table II with the following connotations being used:

	Frequency	*Equivalent loudness*	*Actual time*
For the first impulse	N_1	db_1	T_1
For the second impulse	N_2	db_2	T_2

Data in Table II show that for frequencies of from 50 to 4000 vib./sec. we have one and the same apparent duration independent of the frequency if only the product of the equivalent loudness and time is the same for all. This circumstance makes it possible to omit the frequency in determining the units of apparent duration.

Thus, for a unit of apparent duration we take: the apparent duration of the constant, pure tone which has sounded for one second, at an equivalent loudness of 1 db.

TABLE II.

T_1	db_1	T_2	db_2	T_1db_1	T_2db_2	DIFFERENCE
$N_1=1000$; $N_2=50$						
0.122	43	0.099	52	5.25	5.15	0.10
0.110	63	0.152	50	6.93	7.60	0.67
0.133	80	0.177	59	10.64	10.44	0.20
0.154	87	0.232	58	13.40	13.46	0.06
0.342	46	0.172	90	15.73	15.48	0.25
0.430	41	0.209	83	17.63	17.35	0.28
0.331	84	0.442	65	27.80	28.72	0.93
$N_1=1000$; $N_2=100$						
0.099	85	0.171	50	8.31	8.55	0.24
0.116	87	0.204	50	10.09	10.20	0.11
0.276	49	0.166	84	13.52	13.94	0.42
0.198	85	0.342	50	16.83	17.10	0.27
0.408	53	0.254	85	21.62	21.59	0.03
$N_1=1000$; $N_2=200$						
0.110	75	0.224	38	8.25	7.98	0.27
0.221	49	0.155	70	10.83	10.85	0.02
0.315	43	0.204	65	13.54	13.26	0.28
0.442	43	0.286	65	19.00	18.59	0.41
0.442	55	0.298	80	24.31	23.84	0.47
$N_1=1000$; $N_2=500$						
0.187	51	0.110	82	9.54	9.02	0.52
0.198	63	0.155	84	12.47	13.02	0.55
0.166	97	0.232	71	16.10	16.49	0.39
0.221	87	0.430	45	19.23	19.35	0.12
0.450	51	0.264	82	22.95	21.65	1.3
$N_1=1000$; $N_2=2000$						
0.110	81	0.187	48	8.91	8.98	0.07
0.170	63	0.125	85	10.71	10.62	0.09
0.243	52	0.143	90	12.64	12.87	0.23
0.342	51	0.188	92	17.44	17.30	0.14
0.375	63	0.276	85	23.62	23.46	0.16
$N_1=1000$; $N_2=4000$						
0.155	54	0.098	83	8.37	8.13	0.24
0.209	52	0.133	83	10.87	11.04	0.17
0.188	84	0.320	48	15.79	15.36	0.43
0.232	81	0.386	46	18.79	17.76	1.03
0.413	54	0.265	83	22.62	22.0	0.62

This determination is of significance within limits for which the integral law holds true.

It must be added that the significance of the musical optimum of reverberation lies within these limits, namely, the apparent duration for the optimum, as seen further on, is equal to 41 units (taking the sound threshold as 1.10^{-16} watt/cm^2).

In Fig. 3, the heavy, dotted line indicates the limits for the musical optimum; the upper A for the optimum of sound cinema halls, theaters; the lower B, the optimum for concert halls.

If we go beyond these limits, then we apply the integral law of apparent duration for the determination of the musical optimum reverberation for halls of a capacity up to 80,000 cu. m.

II. MUSICAL OPTIMUM REVERBERATION

To obtain the equation of the dependence of the optimum reverberation on the volume of the hall, it is necessary to equate the integral of the apparent duration of sound for the time of reverberation with the constant value K. This value, as is seen later, remains unchanged irrespective of the volume of the hall, and also irrespective of the type of the hall (whether concert, radio studio, sound cinema, theater, etc.).

We have used the following symbols:

i, normal threshold of audibility;
I, sound intensity;
$I'=I/i$;
$L=10 \log I'$, equivalent loudness;
S, level above threshold.

For the moment when the source of sound is stopped we will use corresponding symbols:

$$I_0,\ I_0',\ L_0,\ S_0;$$

t, time;
t_1, actual time of reverberation (with any I');
T, reverberation time (when $I'=10^6$);
T_{op}, musical optimum of reverberation (when $I'=10^6$);
$A_u=L_0/S_0$, transferred coefficient from equivalent loudness to level above threshold;
K, apparent duration of sound perception;
V, volume of the hall;
v, rate of sound;
a, absorption in the hall;
E, average statistical power for musical performers.

As stated above, we equate the integral of apparent duration of a sound value K'.

$$\int_0^{t_1} \log I'dt=K' \text{ or in decibels } \int_0^{t_1} Ldt=K.$$

For cases of reverberation, I' is the exponential function $I'=I_0'e^{-nt}$. Substituting this and integrating, we get:

$$\int_0^{t_1} \log I'dt=\int_0^{t_1} \log (I_0'e^{-nt})dt$$

$$=(\log I_0'xt_1)/2=K',$$

$$\log I_0'xt_1=2K' \quad \text{but in decibels} \quad L_0t_1=2K.$$

The actual time of reverberation will be to the optimum as the level above threshold is to 60 db.

$$t_1 : T_{op}=S_0 : 60,$$

$S_0 = L_0/A_n$ when $A_n = 1.1$ for 512 vib./sec. This gives us:

$$(T_{op}/60)S_0L_0 = 2K; \quad L_0 = (120KA_n/T_{op})^{\frac{1}{2}}; \quad (1)$$

$$L_0 = (132K/T_{op})^{\frac{1}{2}}; \quad \log I_0' = (1.32K/T_{op})^{\frac{1}{2}}. \quad (2)$$

Let us take Sabine's equation:

$$I_0 = 4E/va; \quad \text{and} \quad T = bV/a.$$

Eliminating a we get $VI_0/T = 4E/vb$. Substituting I_0' for I_0 and T_{op} for T we get $VI_0'/T_{op} = 4E/vbi$. This value is constant for all volumes.

Using Q to express this, we get $VI_0'/T_{op} = Q$. Using this with Eq. (2) we get

$$\log I_0' = \log Q + \log T_{op} - \log V = (1.32K/T_{op})^{\frac{1}{2}}. \quad (3)$$

The equation for the optimum will be as follows for 512 vib./sec.:

$$\log V = \log Q + \log T_{op} - (1.32K/T_{op})^{\frac{1}{2}}, \quad (4)$$

where K is expressed in decibels-seconds, and $\log Q$ in bels.

Thus, in order to be able to obtain the reverberation optimum for halls of a given type, it is sufficient to establish the optimum of reverberation and the mean equivalent loudness of sound of musical performance in a single hall of this type.

Concert Halls

Certain special experiments were conducted in a small hall. Performances of the following soloists and ensembles were tested: pianist, violinist, violin-cellist, baritone and soprano vocalists. For a hall the volume of which was: $V = 290$ cu. m, we found that

$$T_{op} = 1.06 \text{ sec. (at 512 vib. per second).}$$

The mean statistical equivalent loudness for all performances was: $L_0 = 10 \log I_0' = 71$ db; the threshold of audibility being $1 \times 10^{-10} \mu w$.
From (2) we have:

$K = 40.5$ units of apparent duration (db-sec.),

or, roundly, $K = 41$ units.
From (3): $\log Q = 9.534$ bels.

The equation for reverberation will assume the following form:

$$\log V = 9.534 + \log T_{op} = 7.35/T_{op}^{\frac{1}{2}}. \quad (5)$$

This equation is represented by the upper curve A of Fig. 4.

Broadcasting Studios

To determine the optimum time of reverberation in broadcasting studios the following experiments were made: In the broadcasting

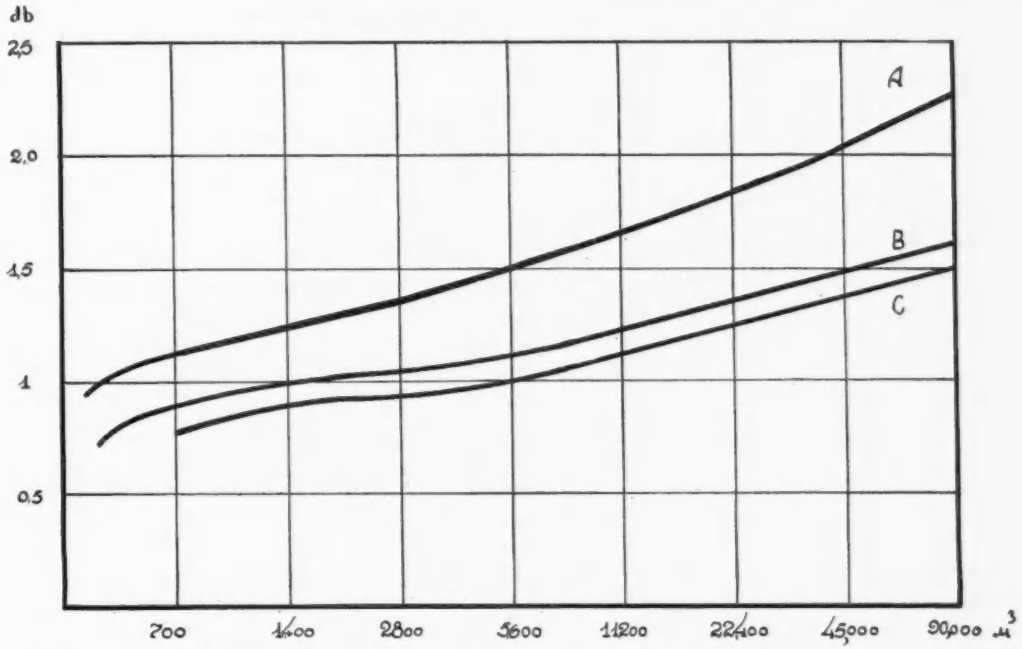

Fig. 4. Optimum of reverberation. A, for the concert hall; B, for the broadcasting studio; C, for the moving picture theaters with sound effects.

studio under test (having x varying absorption) musical performances were tested. These were listened to, in the other studio having complete absorption, by means of the microphone amplifier and dynamic. To eliminate the influence of reverberation of the second studio, the musical performances were listened to at a distance of one meter from the dynamic. In each case the optimum equivalent loudness of this dynamic was properly adjusted. During the musical performance in the first studio the absorption was changed in order to obtain a very distinct optimum. This was determined by the most beautiful quality of the instrument and the voice and the equivalent loudness optimum of the dynamic was then picked up. The following results were obtained:

Studio A of volume $V=180$ cu. m (optimum (T_{op}) $=0.78$ sec.[4] (equivalent loudness $(L_0)=83$ db).
Studio B of volume $=1100$ cu. m ($T_{op}=0.95$ sec.; $L_0=75$ db).

After substituting the data given by studio A, Eq. (2) reads as follows: $K=40.8$.

After substituting the data from studio B, we have $K=40.6$.

The average $K=41$. We may therefore conclude that for broadcasting studios and concert halls the value K is the same, but the value $\log Q$ is different. Indeed, from the data given by studio A we have

$$\log Q=8.3+2.256+0.108=10.664,$$

and from the data of studio B

$$\log Q=7.5+3.042+0.022=10.564.$$

The mean value of $\log Q=10.610\sim 10.6$.

This means that in the case of the reproducer, the optimum equivalent loudness is about 10 db greater than for the usual concert musical reception.

Substituting in Eq. (3) the values of K and $\log Q_1$ we get the following optimum reverberation equation for broadcasting studios:

$$\log V=10.6+\log T_{op}-7.35/T_{op}^{\frac{1}{2}}. \quad (6)$$

This is represented by the middle curve B of Fig. 4. The experimental data obtained for the broadcasting studios coincide with this curve very well.

Moving Picture Theaters with Sound Effects

For such theaters the same condition as for the broadcasting studios holds, that is: $K=41$, and $\log Q=10.6$.

But here another circumstance must be taken into account: the combined reverberation time of electrically coupled rooms. (The combination of the reverberation time of the studio where the film was recorded, and of the reverberation time of the sound moving picture theater.)

The combined time of reverberation of the sound moving picture theater must satisfy Eq. (6).

If we exclude the highly improbable case when the reverberation time of the studio is equal to the reverberation time of the theater, the combined time of reverberation will be about 0.1 sec. greater than the reverberation time of the theater, as is shown in Table III, where the combined time was computed according to Hill.[5]

If we assume the most frequent case to be one for which the combined reverberation rises to 0.1 sec., we obtain the equation of optimum time reverberation for sound moving picture theaters as follows:

$$\log V=10.6+\log (T_{op}+0.1) -7.35/(T_{op}+0.1)^{\frac{1}{2}}. \quad (7)$$

This is represented by the down curve C of Fig. 4. The experimental data are in good agreement with this curve.

Table III.

Studio Vol.	Studio Reverb.	Sound Moving Picture Theater Vol.	Sound Moving Picture Theater Reverb.	Combined Reverb.	Difference
180	0.78	3000	1.05	1.15	0.1
180	.78	14000	1.25	1.35	.1
180	.78	45000	1.40	1.48	.08
1100	.95	3000	1.05	1.25	.2
1100	.95	14000	1.25	1.36	.11
1100	.95	45000	1.40	1.52	.12

[4] The reverberation time of the studio was determined for that part of the studio which was at a sufficient distance from the microphone and for which the values of reverberation (from point to point) change very inconsiderably.

[5] Hill, J. Acous. Soc. Am. **4**, 63 (1932).

Optimum Time of Reverberation for Different Frequencies

From (1) we have:

$$T_{op} = 120KA_n/L_0^2. \quad (8)$$

If, as before, we admit that K has the same value for all frequencies (41 units of apparent duration), the optimum value of different frequencies will be equal to the fraction A_n/L_0^2.

If we are to take L_0 as constant for all frequencies, the optimum of reverberation will be directly proportional to A_n.

The postulate of MacNair:[6] "The velocity of decrease in loudness is the same for all frequencies," and Knudsen's postulate: "All musical components are simultaneously inaudible," are the particular cases for Eq. (8), under the condition that L_0 for all frequencies is the same.

The values A_n can be obtained from the curves of equal loudness, plotted according to Fletcher[7] (loudness level contours).

Fig. 5 shows these curves for equivalent loudness equal to 90–50 db, the limits which occur most frequently during practice. On the average, these curves give the same values for A_n, as shown in Fig. 6 (curve A).

[6] MacNair, Bell Sys. Tech. J., April, 1930.

[7] Fletcher and Munson, J. Acous. Soc. Am. 5, 82 (1933).

The curve A of Fig. 6 represents, also, the reverberation optimum values for different frequencies, the optimum at 1000 vib./sec. being taken as the unit.

If, as usual, we admit the optimum at 512 vib./sec. as the unit, we get curve B of Fig. 6.

If L_0 is in constant for all frequencies, the optimum of reverberation will be proportional to the level A_n/L_0^2.

In Fig. 7 the upper curve A gives the value of the musical optimum of reverberation for various frequencies for concert halls, if we are to consider the average distribution of energy for frequencies for pianos and orchestra, according to Sivian.

The lower curve B gives the optimum of reverberation for various frequencies for vocal auditoriums, with the average distribution of energy in the human voice according to Fletcher.

Conclusions

1. As a unit of apparent sound duration we take the apparent duration of a constant, pure tone which has sounded for one second at an equivalent loudness of one db.

2. The integral law of apparent duration for a constant tone holds true within limits of from 2 to 53 units of apparent duration.

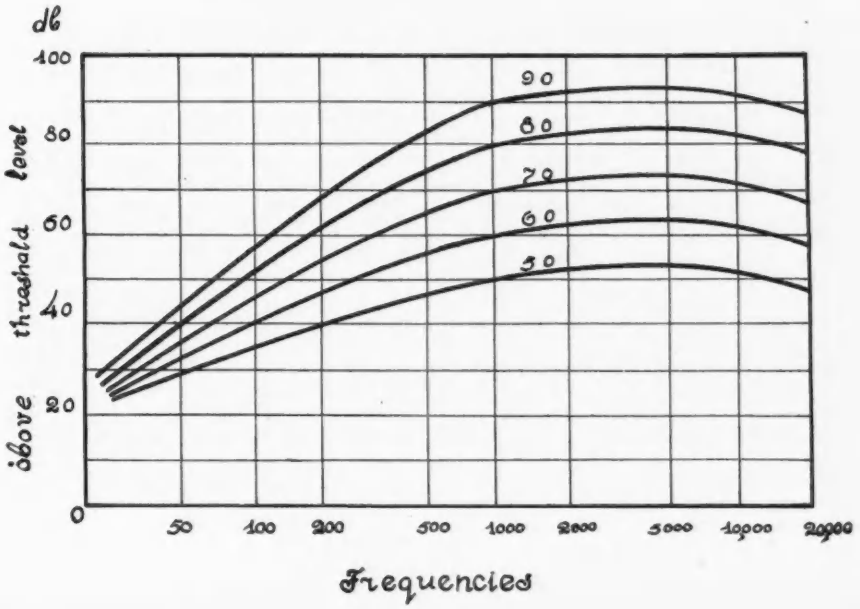

Fig. 5.

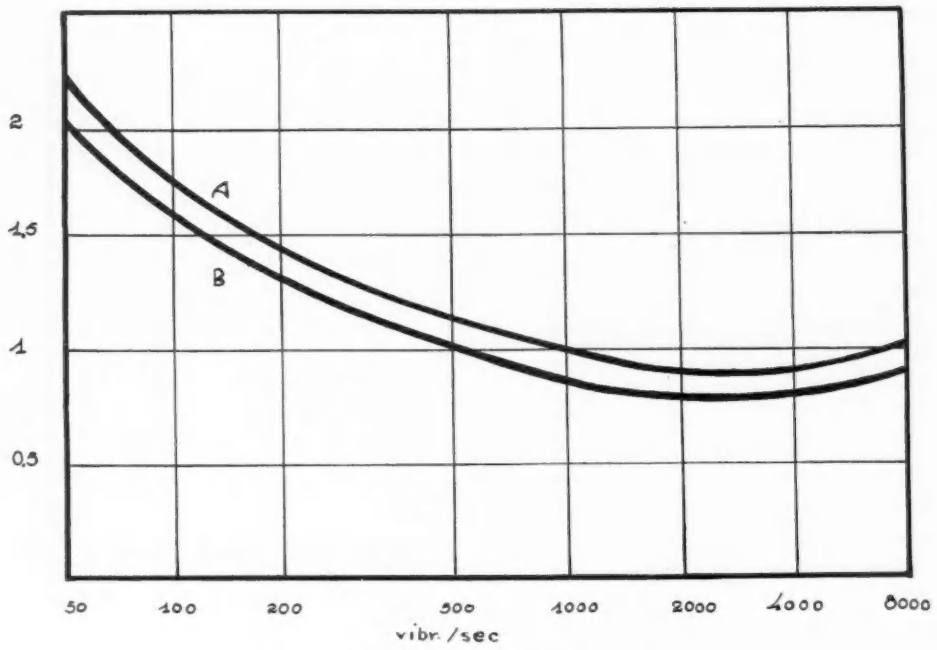

FIG. 6. Values of A_n for different frequencies.

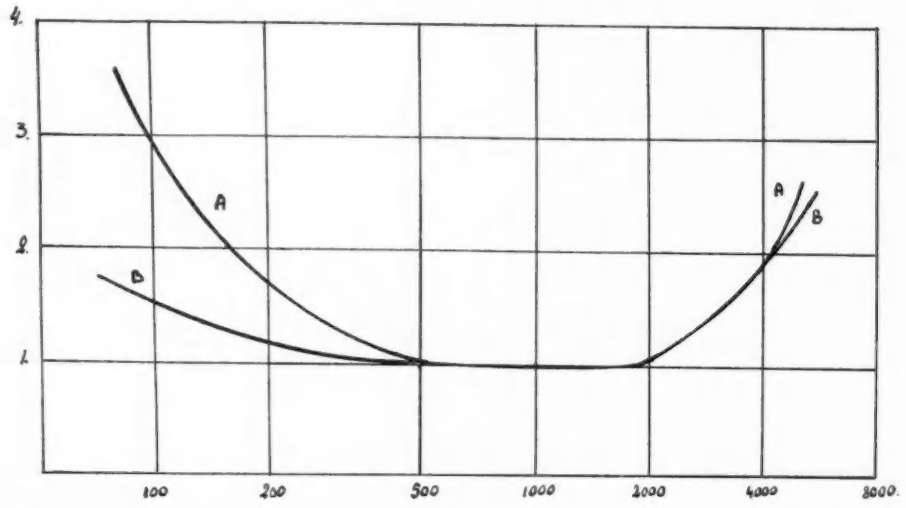

FIG. 7. Optimum of reverberation for different frequencies. A, for music; B, for speech.

3. The integral law of apparent duration holds true for all tones independent of their frequency.

4. The reverberation optimum equations for concert halls, broadcasting studios and sound moving picture theaters may be obtained by equating the duration integral to a constant value.

These equations take the following form for 512 vib./sec.: For the reverberation optimum of concert halls:

$$\log V = 9.534 + \log T_{op} - 7.35/T_{op}^{\frac{1}{2}}.$$

For the reverberation optimum of broadcasting studios:

$$\log V = 10.6 + \log T_{op} - 7.35/T_{op}^{\frac{1}{2}}.$$

For the reverberation optimum of sound moving picture theaters:

$$\log V = 10.6 + \log (T_{op} + 0.1) - 7.35/(T_{op} + 0.1)^{\frac{1}{2}}.$$

5. In all of these cases K is equal to 41 duration units, the value of 1 db$\times$1 sec. being taken as the unit of duration.

6. With the same value of K the dependence of reverberation optimum on the frequency is obtained.

7. The optimum loudness of musical performances at the reverberation optimum for broadcasting studios and sound moving picture theaters is about 10 db greater than the optimum equivalent loudness for concert halls.

I wish to express my indebtedness to S. Skrebkov and A. Galinsky, who participated in the experimental work.

Theory of Transmission of Plane Sound Waves Through Multiple Partitions

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(Received July 24, 1935)

INTRODUCTION

THIS paper presents the results of analyses of a few special cases undertaken by the writer at intervals during the last three years to account for the very high transmission loss of light double elastically supported partitions which from measurements, show a much higher transmission loss than that of a single partition of the same total weight per square foot.

Although only the simple case of plane waves normal to an infinite plane partition is considered, the results seem of sufficient interest to place on record, since the author was unable to find a similar analysis in the literature of acoustics which clearly explained these phenomena.*

It is recognized that in actual cases the sound waves are never perfectly plane and that additional complicated interference effects must arise, but nevertheless the method of treatment used here shows in general, an essential physical reason for the high transmission loss of light flexible multiple partitions.

STATEMENT OF ASSUMPTIONS

These are as follows:

(1) Ideal plane partitions of infinite extent are assumed.

(2) Plane sound waves of normal incidence.

(3) Partitions are perfectly flexible, that is, they contain only mass and have no stiffness of their own.

(4) For compound partitions with air spaces between them the elastic coupling arises only from the layer of air between them, the additional elasticity of flexible supports being relatively small.

(5) Partitions are assumed substantially impervious to air so that leakage effects are neglected.

(6) Partitions are assumed to have perfectly reflecting surfaces.

(7) The air layer between partitions is considered to have a thickness small compared with the wave-length, not much over $\lambda/16$, so this layer may be considered as a lumped elastic coupling, the adiabatic elasticity being used, of course.

(8) Thickness of partitions are small compared with λ for that material.

(9) No sound absorption is assumed, the purpose of this analysis being to ascertain transmission losses apart from damping effects.

Case I. Double partition

Incident wave

$$\dot{y}_1=\dot{y}_{01}e^{j\omega(t-x/c)}+\dot{y}_{01}'e^{j\omega(t+x/c)}.$$

Transmitted wave

$$\dot{y}_3=\dot{y}_{03}e^{j\omega(t-x/c)}.$$

To find transmission ratio $R=y_{01}/y_{03}$, where y_2 and y_3 are the vibration amplitudes of the partitions and m_1 and m_2 are their masses per square foot. k = elastic coupling in pounds per sq. ft. per unit area.

The equations of motion in terms of pressure are

$$\begin{aligned} m_1\ddot{y}_2+ky_2-ky_3&=c\rho(\dot{y}_{01}-\dot{y}_{01}')e^{j\omega t},\\ m_2\ddot{y}_3+ky_3-ky_2&=-c\rho\dot{y}_{03}e^{j\omega t}, \end{aligned} \tag{1}$$

where c = velocity of sound ft. per sec., ρ = mass per cubic ft. of air, in slugs (w/g), m_1 and m_2 are mass in slugs (w/g) per square foot of wall. Note that for a sound wave $p=c\rho\dot{y}$ where p = instantaneous pressure variation from mean atmospheric, and $\dot{y}$ = instantaneous velocity amplitude.

Solving these equations by the usual method we find

$$R^2=\frac{y_{01}^2}{y_{03}^2}=\left\{1-\frac{\omega^2}{2k}(m_1+m_2)\right\}^2+\left\{\frac{\omega}{2c\rho}(m_1+m_2)+\frac{c^2\rho^2}{k}\right)-\frac{\omega^3m_1m_2}{2c\rho k}\Bigg\}^2. \tag{2}$$

If $m_1=m_2=m$

$$db=10\log_{10}\left\{\left(1-\frac{\omega^2m}{k}\right)^2+\left(\frac{\omega m}{2c\rho}\right)^2\times\left(2-\frac{\omega^2m}{k}+\frac{c^2\rho^2}{km}\right)\right\}. \tag{3}$$

* *Note.* Since writing this paper another one by J. R. Constable of the National Physical Laboratory, Teddington, England, published in the Philosophical Magazine, August, 1934, has come to the author's attention which covers the first part of this paper.

It is of interest that our results check exactly when the same restrictions are applied. The expression for transmission loss given by Mr. Constable in Eq. (8) checks that of Eq. (3) of the author's paper when $\omega_0=0$ in Mr. Constable's equation.

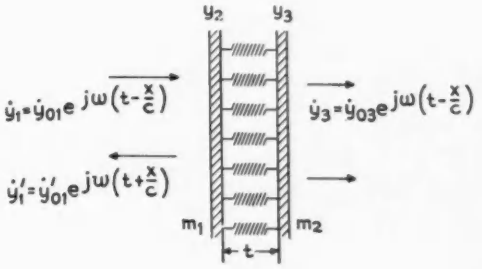

FIG. 1.

If $k=\infty$ and $m'=2m$, (3) reduces to the case of a single perfectly flexible wall

$$db=10\log_{10}(1+\omega^2m'^2/4c^2\rho^2). \qquad (4)$$

For air at atmospheric pressure and temperature, $c\rho$=about 2.6 lbs. per sq. ft. per unit velocity. Expression (4) checks Rayleigh's theory for the same case if the assumed restrictions are applied to his result.[1]

The curves of Fig. 2 and Fig. 3 have been drawn for special cases to which Eqs. (3) and (4) apply.

The frequencies used are rather low and are multiples of 120~/sec. as these represent the frequencies that are sometimes objectionable in electrical engineering work.

Comparing these figures, note that the improvement from the use of the double partition varies greatly with frequency, spacing and weight per square foot.

In general, the double partition is much more effective for the same weight per square foot than the single.

The points of zero T. L. of Fig. 3 for 120~ mean resonance between the two wall layers and the air between them, which has to be looked out for, though in an actual case, wave interference within the partition would largely eliminate this.

A three partition wall would be correspondingly more effective than a two, but subject to more than one such resonance, to be avoided.

Actually, a good double partition wall can be made more effective per lb. per sq. ft. than shown by this theory as we find from noting the results of tests as given in Knudsen's and Sabine's books. This improvement probably arises from the energy absorption introduced in different ways which has been purposely left out of this analysis for simplification and to find to what extent transmission loss could be effected through "isolation" as distinct from absorption.

[1] A. H. Davis, Phil. Mag. (1933).

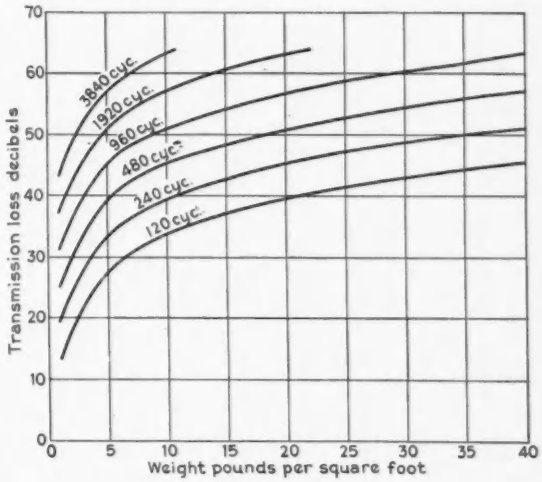

FIG. 2. Transmission loss in single wall due to pure inertia.

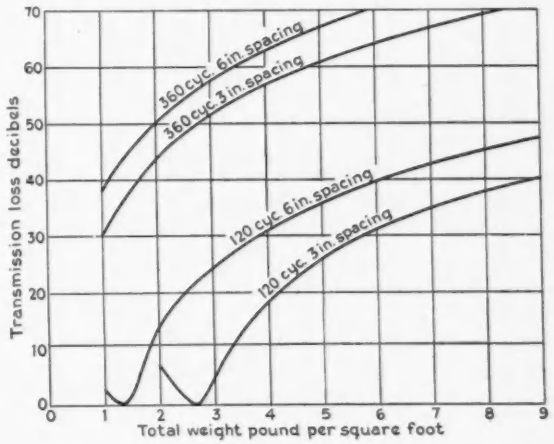

FIG. 3. Transmission loss in double wall with pure inertia.

The isolation of objectionable vibrations in machinery, for example, from transmission to floor structures, is largely a matter of suitable elastic constants for the supports rather than damping which implies frictional dissipation.

Case II. Elastically supported partitions on vibrating wall

$R=y_1/y_3=$ measure of transmission loss.

Setting down the vibration equations of this arrangement (Fig. 4),

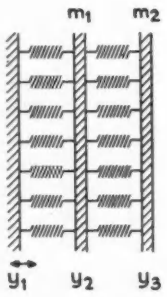

FIG. 4. y_1 = impressed amplitude. y_2 and y_3 are amplitudes of vibrations of the partitions of masses m_1 and m_2, respectively, and elastic coupling constants k_1 and k_2. Note that y_3 is also the amplitude of the radiated sound wave.

$$y_2(k_1+k_2-m_1\omega^2)-y_1k_1-y_3k_2=0,$$
$$y_3(k_2+jc\rho\omega-m_2\omega^2)-y_2k_2=0. \tag{5}$$

These are force or pressure equations which have been directly set down by a method explained in the author's book *Vibration Prevention in Engineering*, Chapter V. This merely sets forth a special way of applying the "mechanical impedance" method of analysis which we have found convenient in our work.

From (5)

$$\frac{y_1}{y_3}=\frac{(k_2-m_2\omega^2+j\,c\rho\omega)(k_1+k_2-m_1\omega^2)-k_2^2}{k_1k_2}. \tag{6}$$

Setting $m_1=m_2=m$ and $k_1=k_2=k$ and eliminating j,

$$R^2=\left(\frac{y_1}{y_3}\right)^2=\left[\left(1-\frac{m\omega^2}{k}\right)\left(2-\frac{m\omega^2}{k}\right)-1\right]^2 + \left(\frac{c\rho\omega}{k}\right)^2\left(2-\frac{m\omega^2}{k}\right)^2, \tag{7}$$

from which the db reduction can be found.

To reduce to the case of a single spring supported flexible wall, let $k_1=\infty$ and $k_2=k$ and $m_2=m$ then (6) becomes

$y_1/y_3=(k-m\omega^2+jc\rho\omega)/k$ or

$$R^2=\left(\frac{y_1}{y_3}\right)^2=\left(1-\frac{m\omega^2}{k}\right)^2+\left(\frac{c\rho\omega}{k}\right)^2, \tag{8}$$

$$db=10\log_{10}\left[\left(1-\frac{m\omega^2}{k}\right)^2+\left(\frac{c\rho\omega}{k}\right)^2\right]. \tag{9}$$

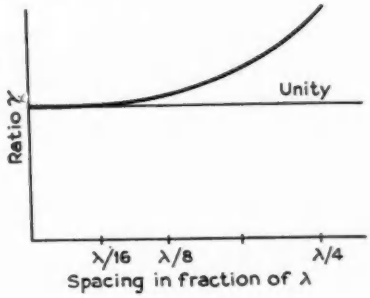

FIG. 5.

The analysis of these cases is of interest because it shows how much reduction in sound radiation from a vibrating wall can be obtained by hanging extra flexible impervious layers on it of certain weight and air space between them. Even one such layer alone shows considerable sound radiation reduction.

There are practical difficulties in the way of applying this method such as lack of ideal flexibility in the outer mass layer, air leakage, etc., but it seems to the author that it offers a possible line of attack on some noise reduction problems where sound is radiated from a surface of considerable area.

It is well known that covering such a vibrating surface with a layer of sound absorbing material is not very effective, in preventing radiation though frequently tried.

LIMITATION OF FORMULAE

As previously noted, Eq. (3) for the double partition requires that the space between these partitions be small compared with the wavelength λ.

Fig. 5 shows the order of error introduced by this assumption, where r equals ratio of true pressure reaction on the second partition to that from the approximate theory here used, as dependent on the space between the partitions expressed as a fraction of λ. This curve was derived by a separate analysis. Thus, the distance t should not be taken over λ/8 for (3) to apply at a given frequency and λ without appreciable error.

It is not a difficult matter to derive the more complete expression taking account of the wave action between the partitions, but the expression is rather complicated for practical use.

The Velocity of Sound in Ethylene Dichloride Vapor

W. H. Billhartz, Jr. and F. L. Bishop, Jr.,* *University of Pittsburgh*

(Received May 6, 1935)

The velocity of sound in ethylene dichloride vapor at the boiling point was determined by the use of a resonance tube. The frequency of the sound was measured by comparison with the frequency of a wire of known length under known tension. Resonance was indicated by minimum reactance of the acoustical impedance of the tube of gas, on the source of sound as the length of the tube was changed by moving a piston. The precision of the apparatus was verified by preliminary measurements with air at 23.5°C and steam at 98.5°C. The values found agree to within 0.5 percent of the results of previous observers. For ethylene dichloride, said to be 99.5 percent pure and so checked by means of its boiling point, the velocity of sound was found to be 176.2 meters per second at 83.7°C.

THE measurement of the velocity of sound in ethylene dichloride $(CH_2Cl)_2$ vapor was undertaken because of the extended use of this substance in industrial chemistry. The velocity of sound is important not only because of its intrinsic interest but also because it furnishes contributing evidence as to the nature of the ethylene dichloride molecule.

Of the many methods of measuring the velocity of sound[1] the one chosen was that in which the gas is confined in a small resonating tube in which the temperature and purity may be accurately established. In the resonance method, one can vary the tube length and hold the frequency of the sound constant or vary the frequency and hold the tube length constant. In this investigation the first method was chosen because a variation in tube length can be measured more accurately and conveniently than can a variation in frequency. A condition of minimum acoustical impedance of the tube of gas was used to indicate resonance. This method required only objective observations. A loudspeaker unit operated by an electrical oscillator served as a source. As the acoustical impedance of the tube of gas changes, the load, and hence the input current to the loudspeaker, varies. Curves showing the variation in loudspeaker current against tube length may be used in determining the wave-length and hence the velocity of sound.

The apparatus used in this investigation is shown diagrammatically in Fig. 1. A tuning-fork 1000-cycle oscillator was used as a source of alternating potential. The amplified output of the oscillator was supplied to the loudspeaker unit. The loudspeaker diaphragm was mounted in front of a baffle plate with a 1.9-cm hole in its center. This unit was placed at the end of a seamless steel tube in which the standing waves were to be set up. The tube was closed at the other end by a movable piston through which four small holes permitted the passage of vapor. At the center of the piston head was placed the junction of a chromel-alumel thermocouple. This piston was moved by a steel screw, having a pitch of 0.100 inch at 20°C. A boiler, made of seamless steel tubing, was used to generate the vapors. This boiler was electrically heated by windings of chromel A ribbon. The vapors, generated in the boiler, passed into the steel jacket in the center of which was mounted the tube in which the standing waves were to be set up. The jacket itself was wound with chromel A ribbon imbedded in "Insulute" and covered with asbestos. The vapors passed the entire length of the jacket, back through the inner tube, and then to a condenser mounted above the other apparatus. A short length of steel pipe returned the condensate to the boiler. The temperature of the vapor in the inner tube was measured with the thermocouple mounted in the piston.

Changes of loudspeaker current caused by the varying acoustical impedance of the tube of gas were measured with a vacuum-tube voltmeter, which consisted of a type "30" tube operated as a grid bias detector. A galvanometer was used to measure changes in the potential drop across a 2000-ohm resistance in the plate circuit. A potentiometer balanced out the main

* Now with American Window Glass Company.

[1] Cf. J. R. Partington, and W. G. Shilling, *The Specific Heats of Gases* (Ernest Ben Ltd., London, 1924).

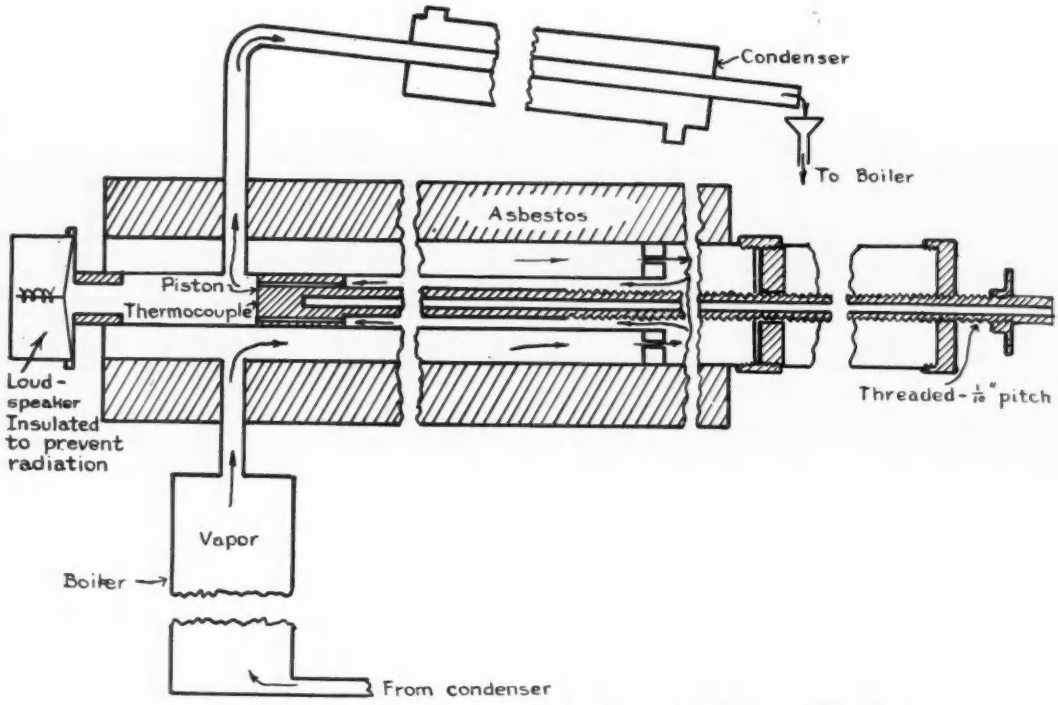

FIG. 1. Apparatus used to measure velocity of sound in ethylene dichloride.

portion of the potential so that a very small change could be measured.

To test the reliability and accuracy of the apparatus, a preliminary test of the velocity of sound in air at room temperature was made. The piston was moved along the tube, and galvanometer readings were taken at 0.5-inch intervals for small deflections and at 0.1-inch intervals for large deflections. Galvanometer deflections were plotted against distance and a curve showing definite peaks resulted. Certain corresponding points of the cycle were determined by extending the slope of each side of the peaks until they intersected. The distances between many pairs of these intersections were recorded and the probable error calculated. The mean of these distances, 6.85 inches ± 0.1 percent, is the half-wave-length in air.

The frequency of the tuning-fork oscillator was determined by comparison with the frequency of a measured length of wire under known tension. The wire was viewed stroboscopically with a neon lamp operated by the amplified output of the oscillator.[2] From the accuracy with which the constant of the wire was known it is estimated that these frequency measurements are accurate to within one part in 2000. The frequency change of the oscillator with a large change of load is not more than 0.1 percent, and the change with temperature is negligible.[3]

The velocity of sound was calculated from the relation $V=\lambda f$, where V is the velocity of sound, λ the wave-length, and f the frequency. The wave-length in air was found to be 13.70 inches ± 0.2 percent. No tube correction was applied to the value since the authors are not aware of any which satisfactorily represents all experimental results of tube corrections.[4] In any case, this correction is very small if the walls are very heavy, as they were in this case. The velocity of sound thus obtained is for air at 23.5°C with a relative humidity of 51 percent. The value obtained was reduced to the corresponding value for dry air at 23.5°C by the following relation:[5]

[2] Cf. R. Hartline, Thesis, University of Pittsburgh (1934).

[3] Information from General Radio Company, Cambridge, Massachusetts.

[4] J. R. Partington, and W. G. Shilling, *The Specific Heats of Gases* (Ernest Ben Ltd., London, 1924).

[5] A. L. Foley, *International Critical Tables*, Vol. 6 (McGraw-Hill, New York, 1926), p. 461.

$$V_d = V_h[1-(e/P)(\gamma_w/\gamma_a-\tfrac{5}{8})]^{\frac{1}{2}},$$

where V_d is the velocity in dry air, V_h in humid air, P the atmospheric pressure, e that part of the atmospheric pressure due to water vapor, γ_w the specific heat ratio for water vapor, and γ_a the specific heat ratio for air. The velocity of sound in dry air at 23.5°C was found to be 344.8 meters per second±0.2 percent. From the *International Critical Tables* the velocity of sound in dry air at 23.5°C is 344.8 meters per second.[6] The close agreement of these two figures is somewhat better than can be expected and establishes the precision of the method.

As a further test of the apparatus, measurements of the velocity of sound in steam were taken in the same manner as for air. To verify that the temperature remained constant throughout the length of the tube, measurements were taken with the thermocouple at various points in the tube. The temperature was found to vary not more than one-half centigrade degree throughout the length of the tube.

The value of the half-wave-length obtained is shown in Table I. This value has been corrected for the temperature expansion of the screw. The velocity was calculated to be 469.6 meters per second±0.2 percent at 98.5°C, and 471.3 meters per second±0.2 percent at 100°C. This value is in very close agreement with the value given in the *International Critical Tables* which is 471.5 meters per second.

TABLE I.

	AIR	STEAM	ETHYLENE DICHLORIDE VAPOR
Mean half-wave-length (inches)	6.85±0.1%	9.30±0.1%	3.50±0.1%
Temperature	23.5°C	98.5°C	81.9°C
Barometer	28.80 in.	28.38 in.	28.73 in.
Relative humidity	51%		
Frequency (cycles/sec.)	992.0	992.0	989.3
Velocity (m/sec.)	348.0	469.6	175.8
Velocity (m/sec.) corrected, see text	344.8	471.3	176.2

In view of the precision indicated by the preceding tests, the apparatus was considered sufficiently reliable to justify confidence in measurements with ethylene dichloride vapor. The procedure in this case was identical with that for water vapor and the results are shown in Table I. The velocity was found to be 175.8 meters per second±0.2 percent at 81.9°C. This yields a velocity of 176.2 meters per second±0.2 percent at the normal boiling point of 83.7°C at a pressure of 76 cm of mercury. The ethylene dichloride used had a guaranteed boiling point range of 83.0°C to 84.0°C which represents a purity of 99.5 percent. The actual boiling point at standard pressure was 83.7°C which is within the limits specified.

The writers wish to express their indebtedness to Dr. Elmer Hutchisson for suggesting the problem and for his assistance during the progress of the investigation.

[6] Reference 5, p. 462.

The Vacuum Tube Oscillator for Membranes and Plates

R. C. COLWELL, *Department of Physics, West Virginia University*

(Received February 21, 1935)

BY USING Chladni's method, it is possible to show the nodal lines on any plate from the position taken up by the sand when the plate is in vibration. The plate is clamped at one point and a violin bow drawn across the edge. In this method it is difficult to produce high notes on the plate or to reproduce any desired figure at will. With some type of vacuum tube it is easy to make a loud steady note and to apply this note to the plate for an indefinite time. The first method which suggests itself is to sound a note on a tuning fork or pipe before a microphone. The resulting electric current is then run through an amplifier. The output of the power amplifier actuates some type of loudspeaker. The best type for this particular purpose is an electrodynamic loudspeaker with a corrugated aluminum diaphragm. To the center of this diaphragm is soldered a three-inch copper rod which has a conical cap. The Chladni plate has a small hole at its center which fits over the cap, supporting and balancing the plate. It is also possible to clamp the plate at any point and apply the vibrating rod to its under side. The same apparatus may be used for membranes except that the membrane can never be balanced upon the cone, but must be held in both hands by its frame and pressed against the vibrating member.

Experiment shows, however, that a musical note sounded before a microphone is likely to vary in pitch and so destroy any sand figure which is forming. The best source of a sustained note is a suitable triode valve. When this valve is placed in the proper circuit, it will produce electrical vibrations varying from several a second to many thousands. When these vibrations pass through a power amplifier and into the loudspeaker, they will set the cone into violent vibration and also produce a musical note.[1]

The membranes used in these experiments are ten inches in diameter or ten inches square. They are made of paper glued on to a wooden frame. Membranes of this size will not form sand figures for high notes because the tension becomes irregular. The sand lines shown in Fig. 1 are for notes lower than five hundred cycles per second.

The brass Chladni plates, if sufficiently thin, will vibrate at much higher frequencies, but another difficulty arises in getting loudspeakers which will vibrate at such frequencies since they are made so as to have the greatest efficiency below five thousand per second. For frequencies above ten thousand per second, a magnetostrictive rod will be found satisfactory. A pure nickel rod inserted in an inductance coil displaces the loudspeaker of the previous set-up. Such a rod will vibrate with a high amplitude at frequencies up to fifty kilocycles a second. The higher the vibrations, the more complicated the figures become, but finally a point is reached at which the plate cannot follow the vibrations of the rod; this is at approximately fifteen kilocycles per second. The sand lines of Fig. 2 were produced with the nickel rod vibrating between twelve and fifteen kilocycles.

In order to produce extremely high vibrations (fifty kilocycles per second), it is necessary to

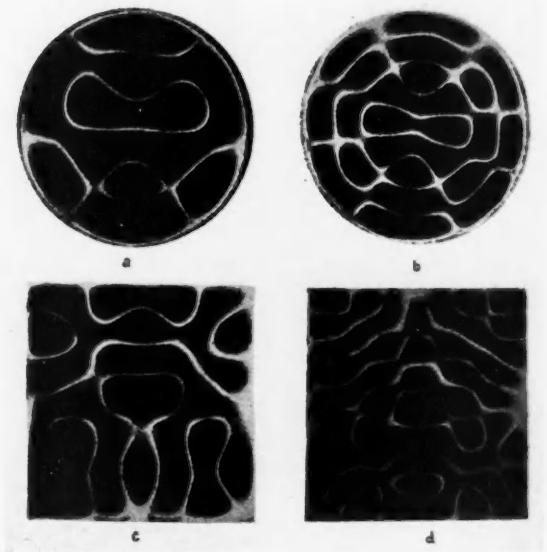

FIG. 1. a, b—nodal lines on circular membranes. c, d—nodal lines on square membranes.

[1] Colwell, Phil. Mag. **12**, 320 (1931).

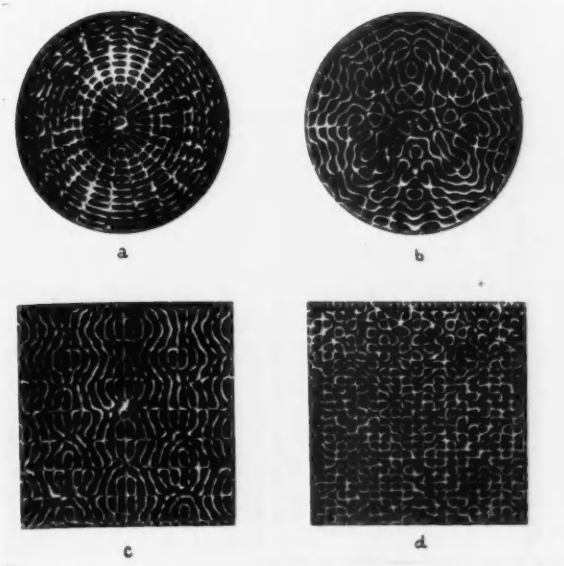

FIG. 2. a, b—nodal lines on circular brass plates.
c, d—nodal lines on square brass plates.

use an ordinary radio sending set. These are usually designed to send out electrical waves varying from fifty to a thousand kilocycles per second. With one of the inductance coils of such a set wound around a nickel rod, it is possible to obtain mechanical vibrations at high frequencies. Since the large Chladni plates will not readily respond to these rapid impacts, thin glass plates one inch in diameter were used. The nodal lines of these plates are shown in Fig. 3.

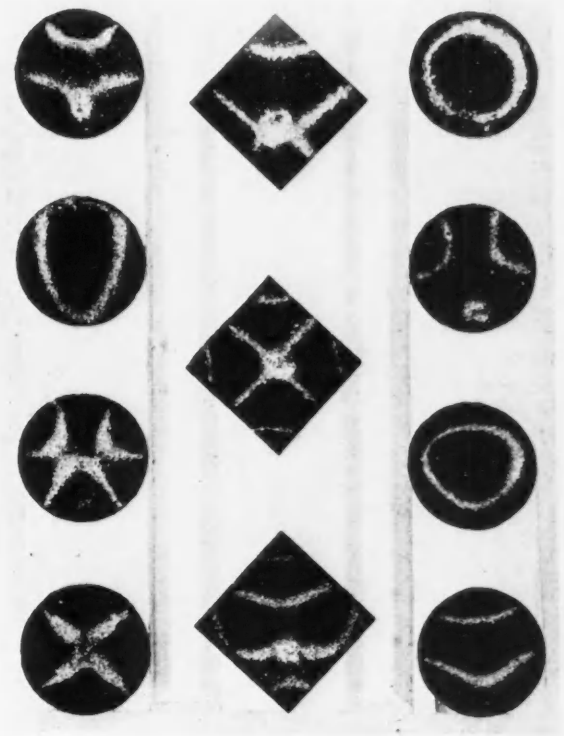

FIG. 3. Glass plates one inch across vibrated at approximately 50 kilocycles.

THEORY

The general theory of membranes and plates is well known but a considerable amount of work is necessary before all the figures can be accounted for. The differential equation of a membrane fixed at the edges is[2]

$$\partial^2 w/\partial t^2 = c^2(\partial^2 w/\partial x^2 + \partial^2 w/\partial y^2). \qquad (1)$$

A particular integral is

$$w = \sin(m\pi x/a)\sin(m\pi y/b)\cos pt, \qquad (2)$$

where

$$p^2 = c^2\pi^2(m^2/a^2 + n^2/b^2) \qquad (3)$$

and m and n are integers.

From these the equation for the nodal lines of a rectangle emerges in the form

$$w = A\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} + B\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b} = 0. \qquad (4)$$

The differential equation for a circular membrane is

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + k^2 w = 0 \qquad (5)$$

and the solution is[3]

$$J_n(kr)\cos n(\theta - \alpha) = 0. \qquad (6)$$

The nodal system is thus divisible into two parts consisting of circles and diameters and combinations of these.

It can be seen, however, from the photographs

[2] Rayleigh, *Sound*, Vol. 1, p. 307.

[3] Reference 2, pp. 319 and 323.

of the actual nodal lines that the curves do not exhibit the symmetries demanded by the conditions. Thus from Eq. (4) all the lines on the square membrane should be similarly related to the four corners and all the curves on the circular membranes should be circles. Rayleigh[4] explains these irregularities by the inequality of tension in the membrane. Another explanation, however, is that the membrane may vibrate in segments as well as a whole. Thus a square may be regarded as made up of two rectangles, each rectangle as made up of triangles and so forth. In this way it is possible to build up many nodal curves which are not symmetrical in the way described above, but which, nevertheless, satisfy Eqs. (1) and (4) for each segment. In addition, A and B may vary on different parts of the plate while m and n remain the same; hence, the plate may divide into two parts showing different curves. The theory of numbers gives many values for m and n which will simultaneously satisfy Eq. (3); therefore, with the same note different parts of the plate may have different nodal lines.

The circle also may divide into semicircles, quadrants, etc., and each of these may have its own pattern so that many curves which are not circles or diameters may be found on circular membranes.

[4] Reference 2, p. 349.

Acoustical Society News

Cambridge Meeting

THE fall meeting of the Acoustical Society was held at Harvard University on December 6 and 7.

The Committee on Arrangement and Program, under the efficient chairmanship of Professor F. A. Saunders of Harvard University, performed its duties excellently. Other members of the committee are: L. S. Ayars, Jr., Johns-Manville, Boston, R. D. Fay, Massachusetts Institute of Technology, F. V. Hunt, Harvard University, W. N. Tuttle, General Radio Company, Cambridge, E. C. Wente, Bell Telephone Laboratories, New York.

The program, printed elsewhere in this issue, lists 27 papers on a variety of acoustic subjects. Commemorative exercises were held in honor of Wallace Clement Sabine in Hunt Hall, where Professor Sabine did his first work in architectural acoustics. The April *Journal* will contain a hitherto unpublished photograph of Professor Sabine, together with an appreciative note by Professor Lyman and an address by President P. E. Sabine. The group of papers on "The Theories of Hearing" brought forth a considerable discussion, and several papers on this subject are planned for the July *Journal*.

Attendance at the sessions varied from 75 to 100 people. Following the dinner on Friday evening, Professor Pierce gave an interesting experimental lecture on "Sounds Made by Insects."

After the Friday afternoon session, tea was served by the wives of the Physics Department faculty, and an opportunity was given for members of the Society to visit the various researches in progress in the laboratory.

Future Meetings of the Society

The next meeting of the Society will be held in Chicago, May 4–5. The fall meeting will come the last part of October in New York City. This change in the usual program of meetings is due to a proposal that the five societies in The American Institute of Physics hold a joint meeting for the promotion of the problems of applied physics. A later announcement giving more details will be sent out by the secretary.

Honor Awarded

"Dr. Edward C. Wente, research physicist in charge of investigations in acoustics in the Bell Telephone Laboratories, New York, received on October 23 the new Progress Medal, which is to be awarded annually by the Society of Motion Picture Engineers in recognition of leading inventions in motion-picture technology. The medal was awarded to Dr. Wente for his work in acoustics and acoustical instruments with special reference to their application to the recording, transmission and reproduction of speech and music." (From *Science*, November 1, 1935.)

Membership List

Part two of the October *Journal* contains a new, up-to-date list of members of the Acoustical Society, arranged both alphabetically and geographically. It gives also a list of officers from the organization in 1929, a history of the Society, and the Constitution and By-Laws. The compiling of the corrected list of members required a considerable amount of work by Secretary Waterfall and Treasurer Stanton.

Deaths

The deaths of the following members are reported: William L. Haven (June, 1934), Allen W. Rowe (May, 1935), Ernst O. Petzold (March, 1935).

Book Reviews

Klänge und Geräusche. F. Trendelenburg. Pp. 235, Figs. 165. Julius Springer, Berlin, 1935. Price 25.8 marks ($10).

Dr. Trendelenburg is a department head in the research laboratory of the Siemens concern in Berlin and also professor in the University of Berlin. In this dual capacity of professor and research director, Dr. Trendelenburg is well qualified to discuss the scientific aspects of acoustics as well as the practical applications. In addition to many articles in special subjects, Dr. Trendelenburg has been interested in writing summaries of the progress of technical acoustics, of which the present work is an extensive example.

Dr. Trendelenburg states that the newer physical methods make possible numerous far-reaching investigations from a critical standpoint of the properties of Klänge (tones) and Geräusche (noises). He discusses not only the physical aspects of the subject, but also the applications in physiology and psychology. The subject is presented in the following divisions: fundamentals of acoustics (Chapter 2) methods of investigation (Chapter 3) closing with applications in the three domains of tone-phenomena of speech (Chapter 4) music (Chapter 5) and noise (Chapter 6). Tone-synthesis and electric music are handled in Chapter 7. Chapter 8 discusses the subjective perception of sound. In Chapter 9, the discussion considers the effects of distortion of tone-perception, which are of fundamental importance in transmission engineering, and in the acoustics of rooms.

The book is up to date, with an extensive bibliography, and 165 figures. The discussions are not limited to the German investigations, with which Dr. Trendelenburg is intimately familiar, but include the researches conducted in the United States and other countries. The book is specially valuable for investigators interested in the practical problems of tone-structure and noise.

F. R. Watson

Sound. An Elementary Textbook in the Science of Sound and the Phenomena of Hearing. F. R. WATSON. 8vo, Pp. 219 + x, Figs. 174. John Wiley & Sons, Inc., New York, 1935. Price $2.50.

The book covers the entire subject of sound, giving the various sections a well-balanced relative importance. The treatment relates to the physical characteristics of the phenomena of sound. Adequate mathematical treatment is introduced to permit the precise statement of quantitative relationships, but the use of higher mathematics for purely theoretical developments is avoided. The book is quite adequate for students of advanced physics who are familiar with elementary mathematics, while at the same time the arrangement of material is such that the general reader and the student of music or psychology may omit certain numerical problems and obtain a satisfactory knowledge of sound. For the more advanced student, references are given for further study.

After a brief account of acoustic phenomena in general, there is an excellent treatment of simple harmonic motion which leads to the study of progressive wave motion and of stationary waves. The importance of simple harmonic motion in the treatment of vibratory and wave motion is maintained throughout. Several chapters deserve special notice because of unusually effective presentation of important aspects of sound. The chapters on the diffraction of sound and on the Doppler effect are especially welcome; these subjects, the practical importance of which is often overlooked, are presented in such a manner as to make them useful. The subject of megaphones is interestingly presented. As one would expect from Dr. Watson, the treatment of sound in rooms is especially satisfactory; the behavior of sound in a room is clearly presented, with the use of numerous diagrams and illustrations which are original with the author; it includes, besides reverberation and interference, the attenuation of sound and its transmission through partitions. The chapter on speech and hearing presents the recent researches on this subject in a very practical manner and more adequately than has been usual in textbooks on sound. A short chapter on the energy in sound waves is excellent.

Other chapters, in their proper sequence, treat of reflection, refraction, interference, Lissajous' curves, air columns, strings, bars, plates, bells, scales, instruments, velocity of sound, and resonance. There is an important and unusual chapter on Experiments in Sound, which, among other things, treats of the vacuum-tube circuit as a source of sound, the tone-variator, the Rayleigh disk, the vibration microscope, the audiometer, and the author's own ripple tank.

At the end of each chapter is a set of problems which are original and effectively formulated to bring out both the philosophical and the quantitative principles of the subject.

There is a list of reference books for advanced study consisting mostly of theoretical textbooks and the most recent treatises presenting the results of research. There are annotations which add much to the usefulness of this list.

The book is abundantly illustrated, and an unusually large proportion of the figures is original and refreshingly new and effective. Mention may be made of the diagrams illustrating refraction (pages 7, 44, 46), interference (pages 52, 53), blurring of speech (page 59), Doppler's principle (page 63), diffraction (pages 72, 74, 80), and acoustics of rooms (pages 148–157). The author's photographs of ripples on the surface of water are exceptionally effective in exhibiting the nature of certain complex wave propagations (pages 53, 63, 72, 197).

The treatise presents a well-balanced and coordinated selection of subject matter with a very satisfactory adjustment as between the historical, experimental, and theoretical aspects of modern acoustics. It is a welcome addition to the rather meager library of works for advanced students of sound.

DAYTON C. MILLER

Noise. N. W. MCLACHLAN. Pp. 148. Oxford University Press, 1935. Price $2.25.

Dr. McLachlan's wide acquaintance with the literature concerning electrical apparatus for generating and measuring sound, together with the associated theory, allowed him to write a satisfactory review on the subject of noise. The bibliography lists the titles of 132 papers, practically all of which have been written in the past five years.

The discussion is divided into ten chapters, which include the following subjects: General considerations: Behaviour of the ear; Measurement of noise; Frequency analysis; Noise in buildings; Traffic noise; Train noise; Noise due to aeroplanes and motor vehicles; Vibration due to machinery; Noise due to electrical machines; Physiological and psychological effects of noise.

This critical survey will be advantageous for investigators in the subject, and should help in the general program for the control of noise.

F. R. WATSON

A Fugue in Cycles and Bels. JOHN MILLS. Pp. 264, Figs. 34. Van Nostrand. Price $3.00.

Herein the acoustical researches of the Bell Telephone Laboratories, quite familiar to the readers of this journal, are expounded in a popular style calculated to be understandable to intelligent music lovers. The physical characteristics of musical sounds, the influence of the facts of hearing on the reproduction and perception of music, the use of acoustical instruments as aids in music teaching, and the application of recent electrical techniques to the production and modification of musical sounds are among the subjects discussed. The last chapter gives a number of graphs in "cycles and bels" pertaining to hearing, noise levels, and acoustical powers of musical instruments.

This book provides an opportunity for musicians to discover the facilities which acoustical science places at their command. Let us hope that it may aid in developing a few more brilliant musical innovators like Dr. Leopold Stokowski.

F. A. FIRESTONE

Practical Solution of Torsional Vibration Problems. (With Examples from Marine, Electrical and Automobile Engineering Practice.) W. K. WILSON. Pp. 438 + xviii, Figs. 106. John Wiley and Sons, New York, 1935. Price $7.00.

The book is written for engineers dealing with design problems in which torsional vibration becomes of practical importance. The design of automobile transmission systems or of the internal combustion engines for marine propulsion, or of electric generating sets direct-coupled to internal combustion engines are examples of such problems. Having in mind practical applications, the author presents the theory of vibration in a very elementary form and in all cases, where it is possible, eliminates the use of calculus and reduces calculations to tabular form.

The first chapter of the book deals with the simplest torsional problems such as two- and three-mass systems. It is shown in numerical examples how the frequencies of free vibrations of such systems can be calculated and also how such calculations can be used in the case of multi-mass systems when only an approximate value of the fundamental frequency of torsional vibration is required.

In the second chapter multi-mass systems are considered and on the basis of Gümbel's work. The frequency tabulation method is described and it is shown how the frequencies of vibration can be calculated with sufficient accuracy by the trial and error method. The method is illustrated by numerous numerical examples dealing with marine installations and direct-coupled generating sets.

The third chapter deals with calculations which are required when replacing an actual complex system by a simpler system consisting of a series of disks attached to a weightless shaft. In this chapter the author goes into all the necessary detailed explanations as to how the actual moving masses can be replaced with sufficient accuracy by equivalent rotating disks and also how a shaft of a variable cross section or a crank shaft can be replaced by an equivalent shaft of a uniform diameter. All necessary information regarding torsional rigidity of crank shafts and empirical formulas based on new experimental work are discussed in this chapter.

In the fourth chapter the author discusses forced torsional vibration of a shaft at nonresonant speeds. In such cases friction can be neglected in calculating amplitudes of vibration and stresses in shafts. The two-mass system is considered first and it is shown that the maximum amplitude of a forced vibration is obtained by multiplying the equilibrium amplitude by a magnification factor which depends on the ratio of the applied frequency to the natural frequency of the system. Afterwards the multi-mass systems are considered by analyzing an actual vibration into a series of normal modes of vibration and by using the principle of superposition.

The fifth chapter deals with forced torsional vibrations at resonant speeds. In such cases damping forces are of importance and the author gives a detailed discussion of these forces. The propeller damping and the damping due to elastic hysteresis are considered in great detail and necessary information and new experimental data are given. These data are used in numerical examples and maximum torsional stresses produced at resonance are calculated.

In Chapter 6 the author describes the methods of experimental determination of the amplitudes of torsional vibrations in various practical cases. A very complete analysis of the known Geiger's torsiograph is given and it is explained how the instrument can be used in various cases and how the records obtained must be analyzed. Several practical cases are discussed with all necessary detail.

Chapter 7 contains the description of various devices for damping vibration. It is shown in examples how these devices can be properly designed. In the last chapter the dynamic characteristics of electrical generating sets direct-coupled to internal combustion engines are discussed.

We must be grateful to the author for collecting very interesting material and presenting it in the form of a book. The book is outstanding because of the simplicity with which it treats the problems and by the number of numerical examples taken from practice and worked out with all the necessary details. It will be useful to designers and can be recommended to engineers dealing with practical torsional problems.

S. TIMOSHENKO
Ann Arbor, Michigan

The Flute (Catalog of Books and literary material relating to the Flute and Other Musical Instruments, with Annotations). DAYTON C. MILLER, Pp. 120. Printed privately by D. C. Miller.

Professor Miller has an interesting collection of flutes of all kinds, some of historical interest, but most of them of scientific value in showing the tone effect of material, shape, air openings, mechanical devices, etc. Professor Miller personally built a gold flute as a pleasurable experiment, to ascertain directly the construction difficulties and to determine the tonal qualities of a flute made of the heavy gold metal.

The Foreword to the publication presents the author's own conception of the work, and is therefore quoted since it gives the most appropriate description of the book.

The making of collections is a highly developed fine art and a science of great antiquity. Probably it originated as a pleasant pastime; often it becomes a serious matter and is carried on systematically for purposes of instruction. The collection of "hobbies" is of infinite variety and brings much diversion, amusement, and information to its devotees.

The history of the art of music is an essential part of the history of civilization. The story of the flute which is perhaps the earliest of musical instruments, is an important

part of the history of music, running like a thread through the fabric from beginning to end. Our present musical scale had its origin in primitive flute music. The flute has had a noble history and it is today one of the important members of the musical family, and is greatly beloved by many for its own beautiful qualities. The writer was led by fate to choose the flute as the subject of collective endeavor.

A comprehensive appreciation of the art of the flute requires, besides a knowledge of music in general, also a knowledge of the physical principles of the flute as a sound producing instrument, of the mechanical devices by which these principles are used, and also an acquaintance with the personality of the performer. An adequate collection must cover these various aspects of the subject.

For many years the writer has made every effort, with the assistance of many friends in various parts of the world, to gather all available material relating to the flute, always proceeding critically and systematically, for the purpose of setting forth the history and development of the modern flute as an essential factor in the fine art of music. This material may be considered as constituting five separate collections. (1) Flutes and flute-like instruments numbering 1250 specimens (1934), indexed and classified under seven different categories; types, makers, places of origin, sources from which secured, etc.; (2) books and literary material relating to the flute and other musical instruments. This collection includes books and pamphlets, short magazine articles, newspaper clippings, concert programs, maker's catalogs and price-lists, index of poetical quotations, patent specifications, novels, etc.; (3) music for the flute, consisting of about 10,000 titles, indexed in four ways, accessions, composers, titles, and instrumentation; this collection is especially rich in 17th and 18th century music; (4) works of art relating to the flute, pictures and sculpture, with indexes; (5) portraits of flutists and composers for the flute; autographs of flutists.

The catalog here presented refers to Collection (2), books and printed material; all of the works listed are actually in the collection. Every work included is considered by the collector to be of importance in relation to the history of the flute. There are many items which so far as known do not exist in any other collection. No available known work of prime importance is omitted; nearly every work is represented by an original exemplar. In a very few instances, perhaps not more than a score in number, in which originals of rare works are known only by one or two exemplars in other collections, the collector has been privileged to obtain photostatic reproductions for inclusion in this collection in order to secure completeness. Instruction books and tutors which describe the instrument are included because of historical interest; but works which are exclusively musical, such as exercises and études are excluded.

There are numerous cross references which will assist in finding all the material relating to a particular subject. There are many annotations; no attempt has been made to annotate the entire collection in a systematic manner; such comments as are recorded call attention to some points of special interest, and to particular facts which will directly aid in the historical researches. In this privately issued edition, a few annotations of personal import have been admitted.

In a collection of works treating of the flute there must of course be much relating to other musical instruments. So far as possible all works treating of any of the wind instruments are included, insofar as they relate to the instruments themselves; however, in the case of instruments other than the flute, books of instruction and literary references have not been included.

The "fate" mentioned in the Foreword which led to the choosing of the flute as the subject of collective endeavor was explained by Professor Miller to the writer as follows: There was a fife in the family used by his father in the Civil War, which young Miller played as a boy. Later, in the college orchestra, he logically selected the flute because of its close resemblance to the fife. Still later, he became interested in comparing the relative musical values of wooden and metal flutes, a procedure which involved the development of the phonodeik. He was then encouraged to extend the investigation to include other musical instruments and vocal sounds. Thus the simple fife was the "fate" that led the way finally to the book on the Science of Musical Sounds. Incidentally, Professor Miller's famous gold flute was financed by money he earned as an expert witness in a suit concerning x-ray burns.

F. R. WATSON

Program of the Fourteenth Meeting of the Acoustical Society of America

DECEMBER 6–7, 1935
Harvard University, Cambridge, Massachusetts

FRIDAY, DECEMBER 6, 1935, 9:30 A.M., ROOM 250, JEFFERSON PHYSICAL LABORATORY

1. Sound Transmission in Straight Tubes. L. B. HAM AND R. D. WATKINS, *University of Arkansas.* (15 minutes).—A theory on sound transmission in straight tubes developed in 1933 in preliminary form by A. Press[1] showed that the radiation of the walls led to a "nonstationary or nonconservative type of motion" defined by the differential equation

$$\frac{\partial^2 p}{\partial x^2} = h\frac{\partial^2 p}{\partial t^2} - k\frac{\partial^3 p}{\partial t^3}.$$

A solution of this equation shows that u_x (particle velocity) and p_x (excess pressure) have no longer a constant phase relation with each other as provided in the simple theory.

The present experiments were undertaken to see whether the phase shifts between u_x and p_x exist as indicated in the above theory. A miniature condenser and a miniature velocity microphone were made, each supported by a rod so that they could be placed at any distance within a four foot stove pipe, $6\frac{7}{8}$ inches in diameter. The microphones were remade later so as to be worked within a seamless brass tubing 8 feet long and $6\frac{7}{8}$ inches in diameter. A loudspeaker was placed just within the tube but not touching it. The tube is supported by rubber.

Phase shifts have been found in all experiments. The phase shifts for all experiments performed appear to be essentially regular for the resonant cases and essentially irregular for the nonresonant conditions. For the resonant conditions tried, the phase shifted through a complete cycle for each half wave-length. Certain pure sine waves between 300 and 600 cycles per second only have been tried.

To establish identity of the phase shifts with the theory, h (velocity coefficient) and k (radiative activity coefficient) must be determined. Work is in progress on this phase of the problem.

[1] A. Press, J. Acous. Soc. Am. **5**, 62A (1933). See also Physik. Zeits. Sowjetunion **5**, 4 (1934).

2. An Effect of High Intensity Sound on Cellulose Fibers. STEPHEN A. BUCKINGHAM, *S. D. Warren Co., Cumberland Mills, Maine.* (10 minutes).—It is found that an effect, known in the paper industry as "hydration," can be produced by the irradiation of an aqueous suspension of cellulose fibers with high intensity sound. Methods of production and detection of "hydration" are discussed. A comparison is given of the magnitude of the effect produced by sound with that produced by conventional methods. The relative efficiency of sound in producing the effect is considered also.

3. The Harmonic Analysis of Consecutive Waves in the Vowel "Ah" in Normal Voices and Before and After Surgical Removal of Nasal Polyps. JOHN BLACK, DEAN M. LIERLIE AND JOSEPH TIFFIN, *State University of Iowa.* (15 minutes).—Oscillograms were obtained of the word *top* as spoken by nine normal male voices and by five voices before and after surgical removal of nasal polyps. Every repetition of the wave pattern from the nine normals, and the first, third, fifth, etc., patterns in the sounds of the operated subjects were analyzed by means of a Henrici harmonic analyzer. A total of approximately 500 waves were considered, the analysis in each case reaching at least to the 25th component, and if necessary to the 45th.

The results justify certain conclusions relative to (a) the nature and importance of the temporal factor and wave-to-wave fluctuations in the vowel; (b) the support of the Helmholtz theory of vowel production; (c) location and variations of the three characteristic regions of energy; (d) probable reasons for certain inconsistent results found in studies of tongue position in vowel formation; (e) effect of removing nasal polyps on voice quality.

4. Selective Amplification Aids to Hearing. V. O. KNUDSEN, L. W. SEPMEYER AND N. A. WATSON, *University of California at Los Angeles.* (15 minutes).—The paper states the requirements for a theoretically ideal hearing aid, and describes the role which selective amplification plays in the design of aids to hearing to approximate these requirements. A description is given of the apparatus designed and built at the University of California at Los Angeles under a grant of the American Otological Society for the purpose of laboratory testing of the effect of selective amplification in aiding hearing. The results of articulation tests made with the apparatus described with specially prepared phonograph articulation records are given for several persons with low frequency impairment of hearing.

5. Echoes at Echo Bridge, Newton Upper Falls, Massachusetts. ARTHUR TABER JONES, *Smith College.* (10 minutes).—One span of this stone arch bridge crosses the Charles River. At the abutment on one side of the river a hand clap is returned as a series of echoes of decreasing intensity. Perhaps the sound waves are roughly focused by the arch, first on one side of the river and then on the other. At any rate, the echoes seem to be produced by reflection from the arch, not from some other object.

When the source is, as usual, on the bank it is to be expected that if an observer in a boat half-way across the river hears any echoes they will succeed each other twice as rapidly as when the observer is near the source. This is not the case. The observer in the boat does hear echoes, but they succeed each other *at the same rate* as when heard near the source. The explanation of this fact has not yet been found.

If both source and observer are near the middle of the

river it is to be expected that the echoes will succeed each other considerably more frequently than when both are on the bank. On three different occasions this was found not to be the case. The echoes heard under these circumstances succeeded each other a trifle *less* frequently than when source and observer were on the bank. The reason seems to involve the fact that the boat is some distance above the center of the arch. The sound reflected from the arch is again reflected from the water before it comes to a focus, and this focus is near the vertex of the arch. From this point it spreads downward, is reflected by the water and again by the arch before the first echo is heard. On one occasion when the water was low the source and observer in the boat were nearer to the center of the arch, and the echoes came with a considerably greater frequency.

6. The Psychophysics of Frequency Modulation. DONALD A. RAMSDELL, *Harvard University.* (20 minutes).—The auditory sensations which result from a frequency-modulated stimulus provide a method for determining certain characteristics of the auditory mechanism. A tone produced by a beat-frequency oscillator was modulated by means of a rotary condenser in order to investigate the critical rate and range of modulation at which apparent fluctuations in pitch become apparent fluctuations in loudness. The rate of modulation for ranges under thirty cycles at which observers cease to detect a change in pitch is a function of the frequency of the stimulus.

At a given frequency, 500 cycles, continuous increase in the rate of modulation shows first a critical point at about eight modulations per second followed occasionally by a second critical point at about fourteen modulations per second, beyond which changes in both pitch and loudness are detectable simultaneously. The first critical point is a linear function of the rate and range of modulation, except for large ranges where the critical rate decreases.

The critical point phenomena can be interpreted in terms of the persistence of vibration of the basilar membrane.

7. Measurement of Speed Fluctuations in Sound Recording and Reproducing Equipment. EDWARD W. KELLOGG AND ADOLPH R. MORGAN, *RCA Victor Company.* (15 minutes).—High quality reproduction of recorded music is possible only if the speeds of recording and reproducing machines are maintained constant within very narrow limits. As improvements are made in these machines greater refinement in measurement of the variations is required in order that the progress may continue.

The development of an improved "Wow-meter" for such measurements is described, and various alternative schemes are discussed. The securing of a standard record, itself sufficiently free from imperfections, presents one of the most serious difficulties. For testing turntables a toothed steel wheel was built, in which special precautions were taken to minimize indexing and other possible errors. The current generated by means of this wheel is supplied to an electrical circuit so arranged that variations in frequency cause deflections of a galvanometer which are registered on a photographic film. The principle employed, namely, a resonant circuit slightly off tune, has been used in earlier devices. In the meter described, a push-pull arrangement has been adopted to reduce the likelihood of errors due to variations in voltage. A frequency meter of this type has certain inherent limitations in its ability to register very rapid fluctuations. Certain measures have been adopted to make the meter satisfactory in this respect, and the possibility of going further in this direction if necessary is given consideration.

8. Flutter in Sound Records. T. E. SHEA, W. A. MACNAIR AND V. SUBRIZI, *Bell Telephone Laboratories.* (40 minutes).—Frequency modulation of a sound signal produces some very interesting and sometimes not readily identifiable auditory sensations. Distortion of this type is caused in sound records by the nonuniform speed of the record during the recording or reproducing process.

While the physical nature of frequency modulation and some of the physiological effects produced by it are discussed, the paper consists largely of demonstrations including a demonstration of the side-band nature of flutter, the influence of room acoustics on the perception of flutter, the auditory effects of flutter in single frequency tones, and a demonstration of the auditory effects of frequency modulation in speech and music.

FRIDAY, DECEMBER 6, 1935, 2:15 P.M.

Hunt Hall

9. Design Compromises for a Practical Sound Meter. ERNEST J. ABBOTT, *Physicists Research Company, Ann Arbor, Michigan.* (15 minutes).—Apparently there is no ideal sound meter, and any design represents a compromise between many factors such as cost, size, weight, sensitivity, range, reliability, and convenience and flexibility of operation, etc. While meters now on the market are reasonably satisfactory for noise surveys and sales demonstrations, few if any of them are suitable for practical machinery noise reduction work.

Some time ago the author was approached and asked to write a specification for a practical sound meter for general purpose noise work. This was an unusual opportunity to allow some one else to do all the development work on the well-nigh impossible meter which I had long hoped to build for my own use, and naturally I agreed. The present paper outlines the design compromises which were made, the reasons therefor, and the meter which was the result of the specification.

10. A Nondirectional Moving Coil Microphone. L. W. GILES, *Bell Telephone Laboratories.* (15 minutes).—This paper describes a moving coil microphone, one of the principal features of which is its nondirectional characteristic, that is, its ability to pick up with equal fidelity sound coming from all directions. This nondirectional

property is attained largely by the use of an acoustic screen which is located in front of the diaphragm of the microphone. In addition, this microphone possesses the advantages of small size, light weight, and a uniform response over a wide range of frequencies.

11. A Compound Horn Loudspeaker. HARRY F. OLSON AND FRANK MASSA, *RCA Manufacturing Company, Camden, N. J.* (15 minutes).—A new type of loudspeaker is described in which a single mechanism is coupled to two horns: a straight axis high frequency horn and a folded low frequency horn. A theoretical analysis of the combined system is given and experimental data are shown which indicate smooth uniform response from 50 to 9000 cycles, and an efficiency of the order of 50 percent over a large portion of this range.

12. Apparatus and Technique for Reverberation Measurements. FREDERICK V. HUNT, *Harvard University.* (15 minutes).—New apparatus has been constructed for obtaining accurate reverberation data. The conventional method of timing the decay to an adjustable threshold is made automatic by a system of relays which control the following sequence: when a predetermined sound level is attained in the sound chamber the source is turned off and an electric timer is started; when the sound level falls to the adjustable threshold the timer is stopped and the source is turned on. If this recurrent cycle is interrupted after ten repetitions the average time for a single decay can be read directly from the timer. The apparatus is entirely a.c.-operated and 80 db of decay can be recorded in the middle audiofrequency range.

Using this apparatus we have investigated the merits of employing a low pass electric wave filter to remove the rapid fluctuations in the rectified output of the microphone amplifier. This is approximately equivalent to the use of a recorder which is just capable of following the average decay rate. Determination of the average decay rate is not affected by the use of the filter but the average deviation from linearity in the decay curves can be reduced by a factor of at least two.

Meeting in Commemoration of Wallace Clement Sabine

3:30 P.M.

Speakers:

13. THEODORE LYMAN, *Director, Jefferson Physical Laboratory*
14. PAUL E. SABINE

SATURDAY, DECEMBER 7, 1935, 9:30 A.M.

Jefferson Physical Laboratory, Room 250

A GROUP OF PAPERS ON "THE THEORIES OF HEARING"

15. Observations Bearing on the Physiology of the Human Cochlea. STACY R. GUILD, *Johns Hopkins Hospital.* (30 minutes).—I have never held, and do not now have, any theory of hearing. There are as yet too many serious gaps in the factual knowledge of the ear to warrant theorizing. My part in this symposium will therefore be limited to the presentation of the observations made in our laboratory at Baltimore, and primarily to those made on the human ear by correlations of the records of hearing acuity with the histologic sections of the temporal bones. The observations that bear on the physiology of the cochlear part of the inner ear have already been published. (The principal paper with respect to high tones was published jointly with Drs. Crowe and Polvogt in the May, 1934, number of the Bulletin of the Johns Hopkins Hospital; and the data bearing on low tones were presented by me at a symposium of the American Otological Society last May and are published in the September, 1935, number of the Annals of Otology, Rhinology and Laryngology.)

Briefly, the evidence indicates localization for high tones (2048 cycles upward), but there is no evidence of localization in man for low tones (up to and including 1024 cycles). For the frequency of 8192 cycles the most important region of the organ of Corti is about 4 to 5 mm from the basal end of the cochlea; for 4096 cycles it is about 8 to 9 mm, and for 2048 cycles it is about 12 mm from the basal end. All tones up to and including 1024 cycles can be well heard when there is total atrophy of the organ of Corti in as much as the whole lower basal turn of the cochlea (this is about one-third of the entire length of the human cochlea). We have been unable to correlate the lesions found in the apical and middle turns with impaired hearing for low tones. No particular form of localization theory is favored by our evidence.

16. Physiological Acoustics; Pitch. H. DAVIS, S. S. STEVENS AND M. H. LURIE, *Harvard University.* (20 minutes).—The electrical potentials generated in the cochleas of guinea pigs were picked up by electrodes in contact with the bone or external membranes of the cochlea and the neck muscles. These potentials were studied as a function of the position of the electrodes and the condition of the organ of Corti within the cochlea in order to determine the specificity of resonance of the basilar membrane to tones of various frequencies, and thereby discover the role of the cochlea in pitch perception.

Audiograms of the electrical response were taken before and after drilling through the cochlear wall and damaging the organ of Corti at various points. The exact location

and extent of each lesion were determined by subsequent microscopic study of serial sections of the cochleas and these were correlated with the losses in hearing revealed by the audiograms. Good correlations were obtained in 20 animals between the boundaries of lesions and sharp departures of the audiograms from normal.

The positions of the resonant areas for tones of various frequencies indicated by these results are confirmed by the locations derived from an integration of the most recent data for differential sensitivity to pitch in human ears. High tones are localized near the basal end of the cochlea, 2000 c.p.s. is at the middle, and the lower octaves are closely bunched toward the helicotrema.

17. Physiological Acoustics; Loudness. S. S. STEVENS AND H. DAVIS, *Harvard University.* (20 minutes).—Functional relationships obtained from studies of the electrophysiology of the auditory mechanism in animals may be used to determine the physical basis of the psychological aspects of hearing. Thus equal magnitudes of electrical potential are generated by the hair-cells of the inner ear when subjected to tones (of different frequency and intensity) which sound equally loud to the human observer, i.e., equal potential contours correspond to equal loudness contours. Therefore, loudness appears to be proportional to cochlear potential. Since end-organ stimulation can reach the brain only via afferent nerves which conduct series of discrete action potentials, loudness must also depend upon the number of fibers involved in the transmission of a volley of action potentials.

The relation of the size of the cochlear potential to the intensity of sound deviates from the Weber-Fechner relationship in the same manner as do the psychological functions. However, at about 60–80 db above threshold the cochlear potential begins to decrease, and a reversible depression of the response, due to overload, is shown by a "hysteresis" effect. If the overload is prolonged, microscopic degeneration of the hair-cells occurs, or if overload is too great the organ of Corti is shaken loose from the basilar membrane.

For a given tone the potential recorded from the scala vestibuli is out of phase by 180° with that recorded from the scala tympani. Therefore the potential drop is across the membrane, or from end to end of the hair-cells. The potential at the round window is in phase with that in the scala tympani, but the potential at the apex differs from the potential in the scala vestibuli by an angle which increases with the frequency of the stimulus. By moving the grid electrode along the outside of the cochlea, a point is found where the potentials which leak out of the two ends (round window and apex) cancel each other by interference.

18. The Position of Pure Tone Stimulation of the Basilar Membrane. JOHN C. STEINBERG, *Bell Telephone Laboratories.* (15 minutes).—The relation between tone frequency and position of stimulation on the basilar membrane has been calculated from data on differential pitch sensitivity and on the density of innervation of the membrane. The calculations involve assumptions concerning the choice of the upper and lower pitch limits of hearing and the choice of tone levels which should be used in obtaining differential pitch sensitivity data. It is shown that for quite different assumptions the positions of stimulation for tones in the range from 1000 to 4000 cycles are not greatly affected. Outside this range the positions depend on the assumptions. It appears that tones of 1000, 2000 and 4000 cycles stimulate, respectively, portions of the membrane about 1/3, 1/2 and 2/3 of its length away from the helicotrema.

Calculations made several years ago by Wegel and Lane from the differential sensitivity data reported by Knudsen gave the positions of a 1000-cycle tone in the middle of the membrane. The differences between the present positions and the earlier ones are due in part to taking account of the density of innervation of the membrane and in part to differences between the low frequency tone data on differential sensitivity reported by Knudsen and that reported later by Shower and Biddulph and used here. The latter data are believed to be more characteristic for low frequency tones.

19. The Relation between Loudness and Masking. HARVEY FLETCHER, *Bell Telephone Laboratories.* (30 minutes).—This paper describes the derivation of a formula for calculating the loudness of any sound from the masking audiogram obtained with the observer immersed in the sound. This formula contains a new function which is dependent only on the masking audiogram. This function was derived from the loudness function $G(L)$. The calculations by this formula are very simple and give results which agree well with those observed.

20. Change of Pitch with Loudness at Low Frequencies. W. B. SNOW, *Bell Telephone Laboratories.* (15 minutes).—The changes in pitch which result from changes in the loudness of low frequency tones were investigated for several frequencies between 75 and 1000 c.p.s. with a crew of 9 observers. The loudness levels covered the range 20 to 120, and both telephone receivers and a loudspeaker were used as sound sources. Fletcher's pitch standard, a pure tone of loudness level 40, was employed.

All consistent judgments gave pitch shifts downward with increasing loudness, but large differences between individuals were found. Two observers perceived no shifts at any frequency or loudness while three others experienced changes greater than 35 percent at the highest intensity. Appreciable changes with time in individual judgments were found. There appeared insignificant difference between data obtained with telephone receivers or with the loudspeaker. A set of contours of equal loudness level, plotted on coordinates frequency and pitch change in percent, is given. These curves show a pitch change small at 1000 cycles, increasing to maximum at a low frequency and decreasing for still lower frequencies. The frequency of greatest shift increased from about 100 cycles at small loudness to about 200 at loudness level 120.

The Appendix describes automatic test equipment that greatly simplified the testing routine and improved its accuracy.

SATURDAY, DECEMBER 7, 1935, 2:15 P.M.

Jefferson Physical Laboratory, Room 250

A GROUP OF PAPERS ON "PUBLIC ADDRESS SYSTEMS"

21. A Method for Obtaining Natural Directional Effects in a Public Address System. R. D. FAY, *Massachusetts Institute of Technology.* (15 minutes).—In studying the effects produced by two or more sources simultaneously emitting the same sounds, it has frequently been noticed that all of the sound appears to come from the nearest source provided the difference in distances between the observer and the sources lies between fairly definite limits. A group address system was set up to utilize this effect in making it appear to the audience that the amplified sound of the speaker's voice came directly from his mouth. Excellent results were obtained, even with relatively great amplification, when the sound reproducing units were about 20 feet behind and 45 feet above the speaker.

22. A Method for Maintaining in a Public Address System the Illusion that the Sound Comes from the Speaker's Mouth. W. M. HALL, *Massachusetts Institute of Technology.* (15 minutes).—As described in the paper by Professor Fay, it is possible to maintain the illusion that the sound comes from the speaker's mouth by arranging the reproducing system so that the original sound reaches the listener's ear before the reproduced. A study has been made of this effect to determine some of its limitations and possibilities.

23. Sound Reenforcing Systems. HARRY F. OLSON, *RCA Manufacturing Company, Camden, N. J.* (30 minutes).—The inadequate speech power of the human voice in theaters and auditoriums is one of the most serious limitations to good hearing. The type of theater which suffers most from inadequate loudness of speech is, of course, the very large enclosed theater and the open air theater. Further in some instances the music furnished by an orchestra is of inadequate volume range to render full artistic interpretation. In these cases means are required for augmenting the intensity of the original sound. The systems for accomplishing this objective are termed sound reenforcing or public address systems. This paper describes the arrangement of the microphones, microphone mixing system and the loudspeakers for increasing the level of the original sound as well as obtaining uniform sound intensity in all parts of the theater. The use of directional microphones and loudspeakers minimizes difficulties due to feed-back. Arrangements for outdoor sound reenforcing systems for various applications are discussed. Announce and call systems are discussed with respect to the arrangement of loudspeakers and microphone locations.

24. Sound Reenforcement—An Acoustic Problem. S. K. WOLF AND G. T. STANTON, *Electrical Research Products, Inc.* (30 minutes).—This paper discusses the approach to the subject of public address as an acoustic problem.

The primary function of a public address system is to faithfully produce, at all points in the listening area, sound of adequate loudness as heard close to its origin. The attainment of this objective is discussed in terms of the acoustic factors involved a d their relation to the physical design of the apparatus.

The first of the factors is the directional-frequency characteristic of the loudspeakers involved, in terms of their relationship to the acoustic transmission of the auditorium. The tremendous complexity of computation of acoustic transmission requires, in the present state of the art, empirical determination of this factor by instrumental means. Since design limitations in the loudspeaker are such as to seldom permit securing exactly the frequency-directional characteristic required, a modification in the characteristic of electrical energy input to the speaker must be made. The requirement of attaining adequate loudness may be limited by acoustic regeneration or "feedback." A brief discussion of the controlling factors is included.

25. Electro-Acoustics. T. F. BLUDWORTH, *New York.* (30 minutes).

26. The Performance of a Public Address System as a Function of Auditorium Absorption. C. P. BONER, *University of Texas.* (10 minutes).—This paper summarizes the work done in connection with public address systems at the University of Texas, particularly in Gregory Gymnasium, an auditorium of one million cubic feet capacity, seating eight thousand people. The results of sound pressure measurements are given, as they show the nature of absorption in the auditorium and the resulting effect on acoustic regeneration and the frequencies at which "singing" takes place. Methods of reducing acoustic regeneration are discussed, which, in the case of Gregory Gymnasium, permit the public address system to establish a sound pressure of approximately twenty bars in the rear of the room, with microphone on the stage and a small orchestra playing.

27. Certain Subjective Phenomena Accompanying a Frequency Vibrato. W. E. KOCK.